Mathematics 103 (Applied Calculus I) Laboratory Manual

Department of Mathematics & Statistics
University of Regina

1$^{\text{st}}$ Edition

by Karen Meagher

History

An electronic version of this document is available at
https://www.uregina.ca/science/mathstat/assets/docs/pdf/M103-LabManual_1ed.pdf

The source of this document (i.e. a transparent copy) is available at
https://www.uregina.ca/science/mathstat/assets/docs/pdf/M103-LabManual_1ed.zip

In the electronic version of this document, page and reference numbers in blue colour, such as those found in the table of contents, and web urls in pink colour as used above are hyperlinks which can be used for easy navigation of this document and connection to the relevant web pages respectively.

Contents

Inspiring Quote

"You could say this advice is priceless", Miss Tick said. "Are you listening?"

"Yes", said Tiffany.

"Good. Now ... if you trust in yourself..."

"Yes?"

"... and believe in your dreams..."

"Yes?"

"... and follow your star..." Miss Tick went on.

"Yes?"

"... you'll still get beaten by people who spent their time woking hard and learning things and weren't so lazy. Goodbye."

From Terry Pratchett's *The Wee Free Men*

Module 1

Equations and Functions

1.1   Linear Functions

1-3: Determine an equation for the line that satisfies each of the conditions below.

1. Contains the points $(-2, 1)$ and $(4, -1)$.

2. Has a slope of 5 and crosses the y-axis at $y = -12$.

3. Has a slope of -2 and contains the point $(2, 1)$.

_____

4. On a single coordinate plane:

   i) plot the three linear functions described below,
   ii) label both the x-intercept and the y-intercept for each line,
   iii) rank the functions by how fast they are increasing,
   iv) determine and label all the points where the lines intersect.

   (a) $f(x) = 3x - 4$.
   (b) $g(x)$ with $g(1) = 5$ and $g(3) = 4$.
   (c) $h(x)$ with slope equal to 2 and with $h(5) = 3$.

1.2   Piecewise Defined Functions

5-8: Express each of the following scenarios as a function. Sketch each function and determine whether or not each function is continuous.

5. In Sunnydale, fines for speeding depend on how fast you are traveling over the posted speed limit. Regardless of speed, each fine has a base fee of $50. If you are caught speeding up to 10 km/h over the limit, you pay $3.00 for each km/h over the limit; if between 10 and 20 km/h the fine increases to $5.00 for each km/h over the limit. Between 20 and 40 km/h over your fine will be $7.50 for each km/h over the limit.

6. In Whoville, your income tax is assessed as follows. If your income is at most $40,000, you pay 15% of your income; from there any income up to $80,000 is taxed at 20% and anything above $80,000 is taxed at 25%.

7. In Cloud City, taxes are structured as follows. If you earn up to $50,000 your taxes are 10% of your income. If you earn between $50,000 and $100,000 you pay a base tax of $5000 plus 12% of your income over $50,000. If you earn above $100,000 you pay a base fee of $10,000 plus 15% on your income over $100,000.

8. In the previous exercise, consider what happens if the base tax of $5000 was changed to $7000 or to $3000? Do either of these changes seem fair? Why or why not?

1.3   Modeling

9-13: For each of the following situations determine a formula which describes it and state the values for which the formula is defined.

9. Let $C(x)$ represent the cost of a speeding ticket in terms of the speed x of a car in a 50 km/h zone. The cost is $100 plus two dollars per km/h over the limit up to a speed of 75 km/h and $150 plus five dollars for every km/h over up a speed of 100 km/h.

10. A farmer has a rectangular field with width w and length ℓ.

    (a) Determine a formula for the area of the field in terms of w and ℓ.

    (b) Determine a formula for the length of fence needed to enclose the field in terms of w and ℓ.

    (c) The farmer plans to section the field with fence as shown in the picture below. Determine the length of fencing needed for this arrangement in terms of w and ℓ.
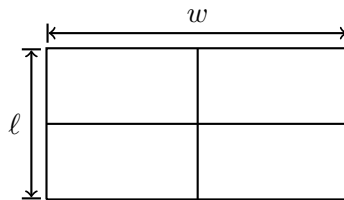
11. A closed box has a square base of side length x and height h measured in centimeters.

    (a) Determine a formula for the volume of the box V in terms of x and h.

    (b) Determine a formula for the surface area of the box in terms of x and h.

    (c) The box is constructed from material that costs $5/cm^2 for the top and $3/cm^2 for the bottom and sides. Determine a formula for the cost of the box in terms of x and h.

    (d) The box is filled completely with solid gold that costs $720/cm^3. Determine a formula for the value of the gold in the box in terms of x and h.

12. You plan to build a window and are considering three different shapes.
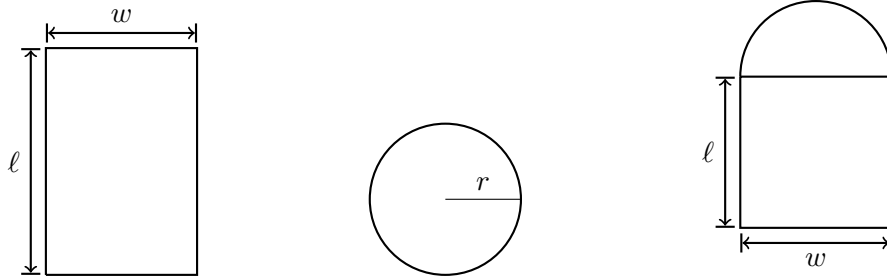
The glass costs $12/\text{cm}^2$ and framing around the window costs $20/\text{cm}$. Determine a formula for the total cost of each window (glass and framing) in terms of the dimensions for the shapes shown above.

13. Let $P(t)$ describe the population of bacteria at time t in hours. Find $P(t)$ if

    (a) Initially the population is 100 and doubles every hour.

    (b) Initially the population is 500 and doubles every 4 hours.

    (c) Initially the population is 1000 and grows by 10% each hour.

1.4   Domains

14-19: State the (real) values of x for which the following functions are undefined.

14. $f(x) = \dfrac{1}{x}$

15. $f(x) = \dfrac{1}{x^2 + 6x + 8}$

16. $f(x) = \dfrac{x^2 - 9}{x - 4}$

17. $f(x) = \dfrac{x - 2}{x^2 + 3}$

18. $f(x) = \dfrac{1}{\sqrt{x^2 - x}}$

19. $f(x) = \sqrt{x^2 + x + 1}$

20-23: For each of the following functions, determine all values of x for which the given function is either zero or undefined. Determine the intervals where the function is positive.

20. $f(x) = x^2 - 7x$

21. $f(x) = 24 - 4x - 4x^2$

22. $f(x) = \dfrac{x + 2}{x - 3}$

23. $f(x) = (x + 4)(x - 2)(x - 5)$

1.5   Sketching Graphs

24-26: Sketch the pairs of functions given below and determine the points where the graphs intersect.

24. $y = \dfrac{1}{x}$ and $y = 2x - 1$

25. $y = 2x^2 + 3x$ and $y = 6 - x^2$

26. $y = \sqrt{x + 4}$ and $y = x - 2$.

1.6   Composition of Functions

27-28: Consider the following questions about composition and evaluation of functions.

27. Let $f(x) = \dfrac{3}{x}$ and $g(x) = x^2 + x$. Determine the following: $f(x)g(x)$, $f(g(x))$, $g(f(x))$, and $f(1)$, $g(1)$, $f(1)g(1)$, $g(f(1))$, $f(g(1))$.

28. Let $f(x) = \dfrac{x-1}{2x+4}$ and $g(x) = x + 2$. Determine the following: $f(x)g(x)$, $f(g(x))$, $g(f(x))$, and $f(3)$, $g(3)$, $f(3)g(3)$, $f(g(3))$.

29-32: Determine the composition $f(g(x))$ for each of the following pairs of functions.

29. $f(x) = \dfrac{1}{x}$ and $g(x) = x + h$.

30. $f(x) = \sqrt{x} + 3$ and $g(x) = x + h$.

31. $f(x) = x^2 + x - 4$ and $g(x) = x + h$.

32. $f(x) = \dfrac{x+1}{x-2}$ and $g(x) = x + h$.

33-34: Determine and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ for the following functions.

33. $f(x) = 2x^2 - 4$.

34. $f(x) = \dfrac{x}{x+2}$.

Module 2

Exponentials and Logarithms

2.1 Exponential and Logarithm Laws

1-4: Simplify the following expressions using the laws of logarithms and exponents.

1. $\dfrac{2^3 4^2}{2^5}$

2. $\ln(x^2 - 4x + 4)$

3. $e^{2\ln(x+3)}$

4. $\ln(e^{2x}e^3)$

5-10: Find all values of x that satisfy the following equations.

5. $e^{3x} = 4$

6. $2 + 2e^{x^2} = 36$

7. $\ln(x^2 + x) = 2$

8. $3 = \dfrac{-2}{\ln(x^2) - 3}$

9. $\ln(x - 2) + \ln(x + 3) = \ln(2x + 6)$

10. $\dfrac{3000}{2 + 4e^x} = 80$

2.2   Applications of Exponentials and Logarithms

11-13: Consider the following questions involving compound interest.

11. Calculate the value of the following investments at the given annual interest rate after the given length of time.

   (a) An investment of \$500 at an interest rate of 6% compounding monthly for 3 years.

   (b) An investment of \$500 at an interest rate of 6% compounding continuously for 3 years.

   (c) An investment of \$1000 at an interest rate of 10% compounding quarterly for 3 years.

   (d) An investment of \$10,000 at an interest rate of 2% compounding daily for 5 years.

   (e) An investment of \$2500 at an interest rate of 10% compounding continuously for 10 years.

12. How long would it take for an investment that is continuously compounding to double when the annual interest rate is 5%?

13. What annual interest rate would be needed for an investment to double in 2 years under continuous compound interest? What interest rate would be needed for an investment to triple in 10 years under continuous compound interest?

   —————————————————————————

14-15: The population of a country usually grows exponentially; this means that the population of a country at time t (years) is given by an equation of the form $P(t) = ke^{rt}$, where r is the rate of growth per year and k is the population at time $t = 0$ years.

14. The population of Nigeria in the year 2010 was $158,423,000$ and it is growing at a rate of 2.79% per year.

   (a) Determine an equation for the population of Nigeria in millions at time t using the 2010 population for $P(0)$.

   (b) What does the equation predict the population of Nigeria to be in the year 2020?

   (c) At the current rate of growth, when will the population of Nigeria reach 200,000,000?

   (d) At the current rate of growth, when will the population double the 2010 population?

15. The population of Malta in the year 1990 was only $354,170$ but had increased to $423,282$ by 2013.

   (a) Determine an equation for the population of Malta (in millions) at time t using the 1990 population for $P(0)$.

   (b) What does the equation predict the population of Malta to be in the year 2020?

   (c) According to the equation, when will the population of Malta be twice what it was in the year 1990?

   (d) At the current rate of growth, when will the population of Malta reach the 2010 population of Nigeria?

   —————————————————————————

Module 3

Limits

3.1 Calculating Limits

1-7: Evaluate the following limits.

1. $\displaystyle\lim_{x\to 3} \frac{x^2+4}{x+2}$

2. $\displaystyle\lim_{x\to -2} \frac{x^2+4x+4}{x+2}$

3. $\displaystyle\lim_{x\to 0} \frac{2-\sqrt{4-x}}{x}$

4. $\displaystyle\lim_{x\to 3} |x-3|$

5. $\displaystyle\lim_{x\to -2} \frac{x^2-3x-10}{x^2-4}$

6. $\displaystyle\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$

7. $\displaystyle\lim_{x\to 1} \frac{x-\sqrt{x}}{\sqrt{x}-1}$

3.2 Limits at Infinity

8-11: Determine the following limits at infinity.

8. $\displaystyle\lim_{x\to -\infty} 3e^x$

9. $\displaystyle\lim_{x\to -\infty} \frac{1}{2-4e^{2x}}$

10. $\displaystyle\lim_{x\to \infty} \ln\left(\frac{1}{x}\right)$

11. $\displaystyle\lim_{x\to \infty} \ln(x^2-x)$

12-15: Determine the following limits. State the degree of the numerator and denominator of each rational function.

12. $\displaystyle\lim_{x\to \infty} \frac{3-x^{26}}{x^5+2}$

13. $\displaystyle\lim_{x\to -\infty} \frac{3-x^{26}}{x^5+2}$

14. $\displaystyle\lim_{x\to \infty} \frac{x^{45}+x^{32}}{2x^{45}-3}$

15. $\displaystyle\lim_{x\to \infty} \frac{3x^4+7x^3}{x^{15}-3x^4}$

9

16. Consider $f(x) = \dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials.

   (a) If the degree of $p(x)$ is strictly smaller than the degree of $q(x)$ what is the limit of $f(x)$ as x goes to infinity or negative infinity?

   (b) If the degree of $p(x)$ is strictly larger than the degree of $q(x)$ what are the two values the limit of $f(x)$ can have as x goes to infinity or negative infinity?

   (c) How may you determine the limit as x goes to infinity or negative infinity if the degree of $p(x)$ equals the degree of $q(x)$?

3.3   Sketching Graphs with Limits

17-22: For each of the following functions:

   (a) Determine the limit at infinity and at negative infinity,

   (b) Determine all vertical asymptotes,

   (c) Determine any intercepts, and

   (d) Sketch the graph of the function.

17. $f(x) = \dfrac{x+3}{x^2+4}$

18. $f(x) = \dfrac{x^3 - 9x}{x^2 + 1}$

19. $f(x) = \dfrac{x^2 - 4}{2x^2 + 4}$

20. $f(x) = \dfrac{3x + 3}{2x - 4}$

21. $f(x) = \dfrac{x^2 - 2x + 1}{2x^2 - 2x - 12}$

22. $f(x) = \dfrac{5x^2 - 3x + 1}{x^2 - 16}$

Module 4

Differentiation

4.1 Difference Quotients

1-3: Determine the composition $f(g(x))$ for each of the following pairs of functions:

1. $f(x) = \dfrac{1}{x} + \sqrt{x} + 3$ and $g(x) = x + h$

2. $f(x) = x^2 + x - 4$ and $g(x) = x + h$

3. $f(x) = \dfrac{x+1}{x-2}$ and $g(x) = x + h$

4-9: Determine the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ for each of the following functions.

4. $f(x) = x + 4$

5. $f(x) = x^3 - x + 4$

6. $f(x) = \dfrac{3x}{1 - 2x}$

7. $f(x) = \ln(x)$

8. $f(x) = e^{2x}$

9. $f(x) = \dfrac{x^2 - 1}{x + 3}$

4.2 Derivatives from First Principles

10-13: Using the definition of the derivative, calculate the derivative of each of the following functions.

10. $f(x) = 3x^2 - x$

11. $f(x) = \dfrac{3}{1 - x}$

12. $f(x) = \sqrt{2 + 4x}$

13. $f(x) = \dfrac{x}{1 + x}$

11

14-15: The function $s(t)$ represents displacement of an object (in meters) at time t (in seconds). Determine the instantaneous rate of change of displacement at the given time.

14. $s(t) = 3t^2 - 4$ at $t = 0$ and at $t = 4$ seconds.

15. $s(t) = \dfrac{2}{t+5}$ at $t = 1$ second.

---

16. Let $f(x) = x^2 + x - 1$.

    (a) Determine the equation of the tangent line of this function at $x = 4$; label this equation $t(x)$.

    (b) Calculate the value of $f(x)$ and $t(x)$ at $x = 4.1$ and $x = 4.2$. What about for $x = 6$? (The point is that for values of x near 4, the values of $f(x)$ and $t(x)$ are close, but for x further from 4 the value of $t(x)$ may be quite different than $f(x)$.)

4.3   Derivatives of Polynomials

17-20: Determine the derivative of each of the following polynomials.

17. $f(x) = 4x^5 - 2x^3 + 3$

18. $f(x) = x^{24} - x^{\frac{2}{3}} - \dfrac{1}{x}$

19. $f(x) = \dfrac{2}{\sqrt{x}}$

20. $f(x) = x^{4.6} + e^2$

21-22: Determine the first, second and third derivative of each of the following polynomials.

21. $f(x) = 20x^3 - 36x^2 + 47x + 454$

22. $f(x) = x^{-4} - \sqrt{x}$

Module 5

Derivatives of Polynomials and Optimization Problems

5.1 Optimization Problems

1-4: Determine the local maximums and minimums and the global maximums and minimums for each of the following functions on the given interval.

1. $f(x) = x^2 - x + 3$ on the interval $[-3, 7]$.

2. $f(x) = x^4 - x^2$ on the interval $[-2, 2]$.

3. $f(x) = 2x^3 + 9x^2 - 168x$ on the interval $[0, 10]$.

4. $f(x) = \dfrac{x^3}{1000} - \dfrac{x^2}{4}$ on the interval $[-100, 500]$.

5-14: Solve the following optimization problems.

5. Find the maximum value of xy with the constraint $x + y = 12$.

6. A farmer wishes to enclose a rectangular section of field at the edge of a straight riverbank using 1000 meters of fenceline. The longer side of the field which borders the riverbank does not require a fence. Express the area of the field enclosed by the fence as a function of the width and determine the dimensions that will maximize the enclosed area.

7. A rectangular field is to be enclosed and then divided into three equal rectangular parts using a total of 32 metres of fenceline. What are the dimensions of the field that maximize the total area of the field?

8. A closed cylindrical can has a radius r and height h measured in centimeters. Express the volume of the can in terms of the radius if the surface area must be 100π cm^2. What radius will produce a can with the greatest volume?

13

9. You need to design a box with no top that has a volume of 100 cm^3 as pictured below.
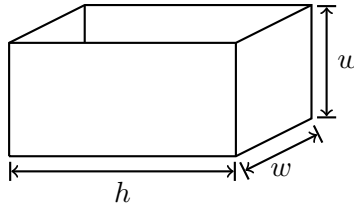
   Determine the dimensions that require the least amount of material for construction of the box.

10. You want to build a storage shed in the shape of a box with a square base having a volume of 100 m^3. The cost of the material to make the base is \$5/m^2 and the cost of the material for the flat roof is \$3/m^2; the material for the sides will cost \$2/m^2. Determine the dimensions of the shed that will minimize the overall cost.

11. A rectangular poster contains 25 cm^2 of printed area surrounded by margins of width 2 cm on each side and 4 cm on the top and bottom. Express the total area of the poster as a function of the width of the printed section and determine the width that will maximize the total area of the poster.

12. A box with no top is to be made from a square piece of cardboard which measures 18 cm by 18 cm. To make the box, a square will be removed from each corner of the cardboard, and the resulting flaps will be folded up. What are the dimensions of the squares that produce a box with the greatest volume?

13. A cylindrical can is to hold 70π cm^3 of frozen juice. The sides are made of cardboard which costs \$0.03/cm^2; the top and bottom are made of metal that costs \$0.06/cm^2. Express the cost of constructing the can as a function of the radius of the can and determine the radius that will minimize cost.

14. A pharmacist wants to establish an optimal inventory policy for an antibiotic that requires refrigerated storage. The pharmacist expects to sell 800 units of this antibiotic at a constant rate during the next year and will place several orders of the same size equally spaced throughout the year. Each order has a delivery fee of \$16 and the carrying cost is on average \$4 per year for one package.

   (a) Let x be the order quantity and t the number of times the antibiotic is ordered in a year. Determine the inventory cost (ordering cost plus the carrying cost) in terms of x and t.

   (b) Determine the order quantity that minimizes the inventory cost and calculate the minimum inventory cost.

Module 6

Chain Rule and Related Rates

6.1   Chain Rule

1-4: Using the chain rule, determine the derivative of each of the following functions.

1. $f(x) = (4x^{34} + 2x^{21} + x)^{-27}$

2. $f(x) = \dfrac{3}{\sqrt{x^5 + 3x^3 - 2}}$

3. $f(x) = (2 + \dfrac{1}{3x})^{12}$

4. $f(x) = (1 + \sqrt{4x})^3$

5-6: Use the chain rule to determine the following rates of change.

5. If $f(r) = r^3 + 3r^2$ and $r(t) = \sqrt{t} + t - 30$, determine $\dfrac{df}{dt}$.

6. At the Happy Unicorns' Candy Factory the cost of making c kilograms of rock candy is given by the formula $f(c) = 34c - \sqrt{c}$. The amount of candy that can be made in t hours is given by the formula $c(t) = 35t + \dfrac{2}{t}$. Determine the rate of change of the cost of making the candy over time, $\dfrac{df}{dt}$.

6.2   Implicit Differentiation

7-8: Evaluate $\dfrac{dy}{dx}$ for each of the following curves and determine the equation of the tangent line at the given point.

7. $x^2 + y^2 - 2 = y + x$ at the point $(1, 2)$.

8. $\dfrac{1}{x + y} = -x + y^2$ at the point $(-1, 4)$.

6.3 Related Rates

> **Recall:**
> The area of a circle in terms of its radius r is given by the formula $A = \pi r^2$.
> The volume of a sphere in terms of its radius is given by $V = \dfrac{4}{3}\pi r^3$.
> The volume of a cone is given by $V = \dfrac{1}{3}\pi r^2 h$ where h is the height of the cone and r is the radius of the base of the cone.

9-15: Answer the following related rates questions.

9. An oil spill takes the shape of a circle. When the spill has a radius of 50 meters its radius is growing at a rate of 0.5 m/hour. How fast is the area of the spill increasing at this time?

10. A spherical balloon is inflated at a constant rate of 5 cm^3/minute. How fast in the radius of the balloon changing when $r = 10$ cm? What about when the radius is 5 cm?

11. A 5 meter long ladder is leaning against a vertical wall. The foot of the ladder starts to slide horizontally away from the wall. At the instant the foot of the ladder is 3 m from the wall the top is falling downward at a rate of 1 m/second. How fast is the foot of the ladder moving away from the wall at this moment?

12. A cylindrical drinking straw has a radius of 0.5 cm. The straw is in a glass of water of radius 5 centimeters. Water from the glass is being sucked up the straw and the the height of the water level in the straw is increasing at a rate of 2 cm/second.

    (a) What is the rate of change of the volume of water in the straw?

    (b) At what rate is the height of the water in the glass decreasing?

13. A cone shaped cup has a height of 10 centimeters and radius of 4 cm. If coffee is being poured into the cup at a rate of 1 cm^3/s, what is the rate of change of the depth of the coffee when the depth is 5 cm? What about at 8 cm?

14. Highway 2 runs north to south and crosses Highway 1 at Moose Jaw. Dashiell is driving his car north along Highway 2, while Ada is driving east along Highway 1. At the point when Dashiell is 10 kilometers to the north of Moose Jaw, he is traveling at a speed of 80 km/h. At the same time Ada is traveling at 100km/h and is 15 kilometers to the east of Moose Jaw. At what rate is the distance between the two cars changing at this moment?

15. A plane is flying at a speed of 750 km/h and at an altitude of 5 kilometers above your head. How fast is the plane's distance from you increasing at the moment when the plane is flying over a point that is 6 km from your position on the ground?

Module 7

Product and Quotient Rule and Graphing

7.1 Product and Quotient Rule

1-2: Calculate the derivative of the following functions using the quotient rule.

1. $f(x) = \dfrac{1}{2x^4 - 3}$

2. $f(x) = \dfrac{5}{2x - 3}$

3-4: Calculate the derivative of the following functions using the chain rule.

3. $f(x) = \dfrac{1}{2x^4 - 3}$

4. $f(x) = \dfrac{5}{2x - 3}$

5-8: Calculate the derivative of the following functions.

5. $f(x) = (x^5 - 5x^4 + x^2 + 4)(x^6 - 4x^4 + x + 2)$

6. $f(x) = (3x + 4)^{23}(3x - 3)^{12}$

7. $f(x) = \dfrac{x^2 + 4}{x + 2}$

8. $f(x) = \dfrac{(x + 1)^2(3x - 1)^3}{x + 2}$

7.2   Sketching Asymptotes

> **Recall:**
> A function is usually **undefined** at a vertical asymptote. A function can cross a horizontal asymptote and the horizontal asymptote indicates what happens to the function as x approaches infinity or negative infinity.

9-14: Sketch a graph $f(x)$ that has a vertical asymptote $x = 1$ and a horizontal asymptote of $y = 0$ that also satisfies the given condition:

9. $f(x) > 0$ for all x.

10. $f(x) < 0$ for all x.

11. $f(x) > 0$ for $x < 1$; $f(x) < 0$ for $x > 1$.

12. $f(x) < 0$ for $x < 1$; $f(x) > 0$ for $x > 1$.

13. $f(x) < 0$ for all $x < 0$;
    $f(x) > 0$ for all $0 < x < 1$ or $x > 1$.

14. $f(x) < 0$ for all $x < 1$ or $1 < x < 2$;
    $f(x) > 0$ for all $x > 2$.

7.3   Graph Sketching

15-20: Sketch the graphs of the following functions using the following steps:

  (a) Determine the x-intercepts and the y-intercept, if any.

  (b) Determine any vertical asymptotes.

  (c) Determine any horizontal asymptotes.

  (d) Determine the intervals of increase and decrease.

  (e) Determine all local maximums and minimums.

  (f) Determine the intervals of concavity.

  (g) Determine any inflection points.

Then produce a clear labeled sketch of the graph using the information from parts (a) to (g).

15. $f(x) = x^5 - 5x^4 + 93$

16. $f(x) = x^3 - 3x^4$

17. $f(x) = \dfrac{x^2}{x+2}$

18. $f(x) = \dfrac{x-3}{x^2+7}$

19. $f(x) = \dfrac{x^2-9}{x^2+1}$

20. $f(x) = \dfrac{(x+3)(x-4)}{x^2-4}$

Module 8

Derivatives of Exponentials and Logarithms

8.1 Derivatives with Logarithms and Exponentials

1-4: Determine the derivatives of each of the following functions.

1. $f(x) = e^{4x^{23} - x^4}$

2. $f(x) = xe^{2x}$

3. $f(x) = \dfrac{e^{-x}}{x+1}$

4. $f(x) = (x^3 + e^x + 3)^4$

5-10: Determine the derivatives of each of the following functions.

5. $f(x) = 2\ln(x^2 - 4)$

6. $f(x) = \ln(2x)$

7. $f(x) = x\ln(x)$

8. $f(x) = (\ln(x) - 5)^3$

9. $f(x) = \dfrac{x^2 - x}{\ln(x)}$

10. $f(x) = x\ln x - (x^2 - x)e^{2x}$

11-12: Determine $\dfrac{dy}{dx}$ using logarithmic differentiation.

11. $y = x^{3x+1}$

12. $y = (2x^2 - 4)^{\frac{1}{x}}$

8.2 Applying Derivatives of Logarithms and Exponentials

13. It is canker worm season in Regina! The population of canker worms triples every week and the population continues to increase at this constant rate. On the initial day of the investation there are 50 worms in the big beautiful tree in front of my house.

19

(a) Determine an equation for the number of worms in my tree as a function of time t in weeks.

(b) Calculate the number of worms in my tree six weeks after the initial day of investation.

(c) At what rate is the population of worms increasing initially?

(d) At what rate is the population of works increasing six weeks after the initial day of investation?

14-17: Using the steps in Section 7.3, sketch each of the following functions.

14. $f(x) = \dfrac{10}{1 + 100e^{-x}}$

16. $f(x) = (\ln(x))^2$ for $x > 0$

15. $f(x) = x\ln(x)$ for $x > 0$

17. $f(x) = 10xe^x$

———————————————————————

18-21: Optimization Problems.

18. Determine the maximum of $\ln(x) - x^2$ for $x > 0$.

19. Determine the local maximum and minimum of $f(x) = x^2 e^x$.

20. The value of an investment is given by $30e^{0.05t}$ (dollars) and the investment brokers fees are determined by $\dfrac{1}{t}$ times the value of the investment at time t. At what time will the brokers fees be at a minimum? What is the fee at this time?

21. The effectiveness of studying for a calculus test depends on the number of hours a student spends studying for the test. From experimental evidence, the effectiveness of studying after t hours is given by $E(t) = \dfrac{2}{5}\left(9 + te^{\frac{-t}{10}}\right)$. A student has up to 20 hours to study for the quiz. How many hours are needed for maximum effectiveness?

———————————————————————

22. There is a rumour afloat at your university that your calculus professor secretly works as a clown at children's birthday parties on the weekend. At time t (in hours) the number of students that know this rumour is given by the equation

$$C(t) = \frac{10,000}{1 + 500e^{-\frac{t}{10}}}.$$

(a) How many students know the rumour at time $t = 0$?

(b) Everyone will eventually hear the rumour. What is the population of the university?

(c) How long until 2000 students have heard the rumour?

(d) When is the rumour spreading the fastest?

(e) Graph this function using a graphing calculator. (Functions of the form

$$f(t) = \frac{c}{1 + ae^{-bt}}$$

are called *logistic functions* and they model the growth pattern of some phenomena.)

Module 9

Integration

9.1 Antiderivatives

1-6: Determine an antiderivative for each of the following functions.

1. $f(x) = 3x^2 - 5x + 6$

2. $f(x) = \dfrac{x^3 + 4}{x^2}$

3. $f(x) = \sqrt{x} + \dfrac{2}{\sqrt{x}}$

4. $f(x) = \sqrt[3]{x^2} - 4x^6 + e^2$

5. $f(x) = \dfrac{3}{x}$

6. $f(x) = e^{2x}$

7-10: Simplify then evaluate each of the following indefinite integrals.

7. $\displaystyle \int \dfrac{4x^3 + 5\sqrt{x} - 4x}{x} \, dx$

8. $\displaystyle \int (x+3)(x-4) \, dx$

9. $\displaystyle \int x\sqrt{2x} \, dx$

10. $\displaystyle \int \sqrt{x}(\sqrt{x} + \sqrt{2}) \, dx$

9.2 The Definite Integral

11-14: Compute each of the following definite integrals.

11. $\displaystyle \int_0^2 3x + 6 \, dx$

12. $\displaystyle \int_0^3 (2x - 4) \, dx$

13. $\displaystyle \int_{-1}^{10} e^x - x \, dx$

14. $\displaystyle \int_1^{e^2} \dfrac{1}{x} \, dx$

9.3   The Substitution Rule

15-19: Evaluate each of the following indefinite integrals using the substitution rule.

15. $\displaystyle\int \frac{x^2 + 2x}{(x^3 + 3x^2 + 4)^5}\, dx$

18. $\displaystyle\int 2xe^{x^2-4}\, dx$

16. $\displaystyle\int (5x+1)\sqrt{5x^2 + 2x}\, dx$

19. $\displaystyle\int \frac{4x - 4}{2x^2 - 4x}\, dx$ (with $x > 0$)

17. $\displaystyle\int x\sqrt{4x+1}\, dx$

_____

20-22: Evaluate each of the following definite integrals using the substitution rule.

20. $\displaystyle\int_0^2 x^3\sqrt{x^4 + 9}\, dx$

22. $\displaystyle\int_1^3 \frac{x}{(2x^2 + 1)^2}\, dx$

21. $\displaystyle\int_0^5 xe^{x^2}\, dx$ (with $x > 0$)

_____

9.4   Areas Under a Curve

23-25: Using definite integrals, determine answers to the following questions.

23. Calculate the area bounded by the x-axis and the y-axis and the curve $y = 5 - x^2$.

24. Let $g(x) = 1 - 2x$ and $f(x) = x^2 + 10x + 21$.

   (a) Graph $g(x)$ and $f(x)$.
   (b) Determine where $g(x)$ and $f(x)$ intersect.
   (c) Calculate the area bounded by $g(x)$ and $f(x)$.

25. Consider the curves $f(x) = e^x$ and $g(x) = 20 - e^x$.

   (a) Graph $g(x)$ and $f(x)$.
   (b) Determine where $g(x)$ and $f(x)$ intersect.
   (c) Calculate the area bounded by $g(x)$ and $f(x)$ and the y-axis.

_____

9.5   Average Value of a Function

26-27: Consider the average value of the function given in each of the following situations.

26. The value of a commodity fluctuates over time and is given by the function

$$f(t) = \frac{1}{100}t^3 - 71t^2 + 1450t - 8325$$

where t is time in years. What is the average value of the commodity from $t = 0$ and $t = 60$?

27. The demand for long-stemmed roses is given by the function $f(t) = \dfrac{5000 \ln(t + 11) - 3}{t + 11}$ where t is the number of days from January 1st. What is the average demand for long stemmed roses over a year (the period from $t = 0$ to $t = 365$)?

_____

Answers

1. $3y + x = 1$
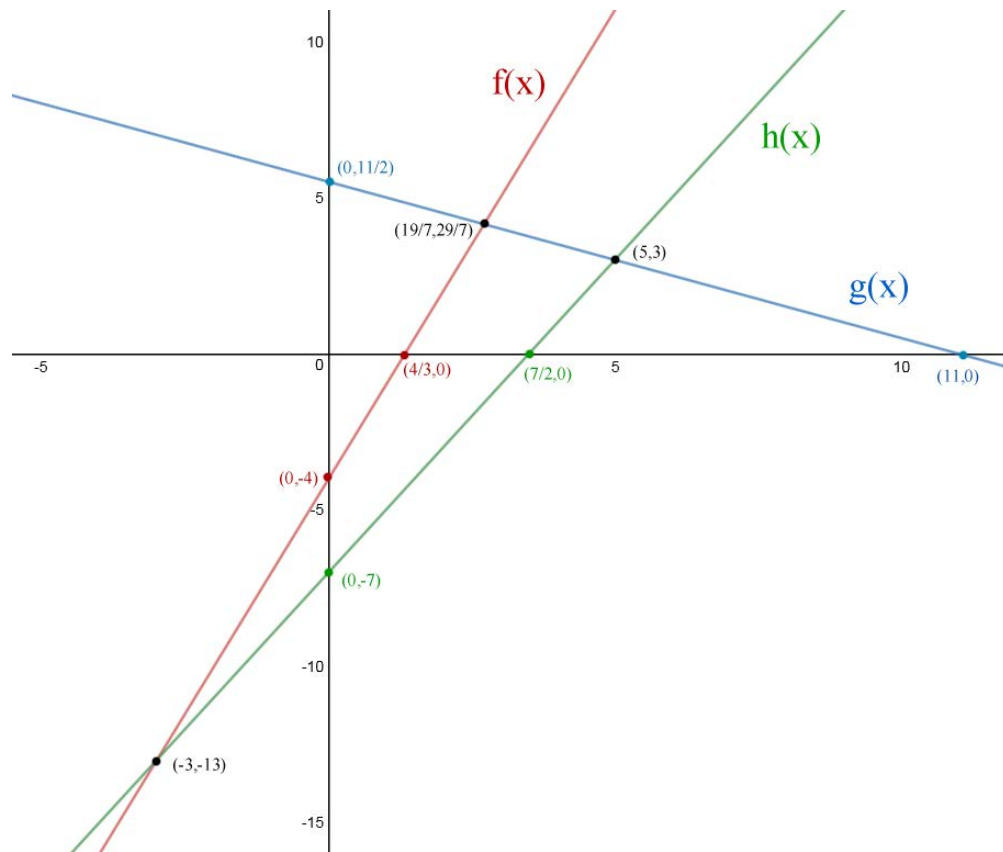   3. $y = -2x + 5$

2. $y = 5x - 12$

4. The linear functions have equations

    (a) $f(x) = 3x - 4$

    (b) $g(x) = -\dfrac{1}{2}x + \dfrac{11}{2}$

    (c) $h(x) = 2x - 7$

See graph for intersection points:

24

5. If x represents speed then the fine (in dollars) is,

$$F(x) = \begin{cases} 0 \text{ if } x \le 50 \\ 50 + 3(x - 50) \text{ if } 50 < x \le 60 \\ 80 + 5(x - 60) \text{ if } 60 < x \le 70 \\ 130 + 7.5(x - 70) \text{ if } 70 < x \le 90 \end{cases}$$

6. If x represents your income then your taxes (in dollars) are,

$$T(x) = \begin{cases} 0.15x \text{ if } x \le \$40,000 \\ 0.2x \text{ if } \$40,000 < x \le \$80,000 \\ 0.25x \text{ if } \$80,000 < x \end{cases}$$

7. If x represents your income then your taxes (in dollars) are,

$$T(x) = \begin{cases} 0.10x \text{ if } x \le \$50,000 \\ 5000 + 0.12(x - 50,000) \text{ if } \$50,000 < x \le \$100,000 \\ 10,000 + 0.15(x - 100,000) \text{ if } \$100,000 < x \end{cases}$$

8. If the base tax changed from \$5000 the graph would be discontinuous; there would be a *jump* from one tax bracket to the next. If the base was \$7000, a person earning \$50,001 would pay an additional \$2000 compared to a person earning \$49,999. In other words, a difference in income of just two more dollars could mean a difference in your tax assessment of \$2000! Similarly if the base was \$3000.

9. $C(x) = \begin{cases} 0 \text{ if } x \le 50 \\ 100 + 2(x - 50) \text{ if } 50 < x \le 75 \\ 150 + 5(x - 75) \text{ if } 75 < x \le 100 \end{cases}$

10. (a) $A = \ell w$ defined for ℓ and w positive.

(b) $F = 2(w + \ell)$ defined for ℓ and w positive.

(c) $F = 3(w + \ell)$ defined for ℓ and w positive.

11. (a) $V = x^2 h$ defined for x and h positive.

(b) $SA = 2x^2 + 4xh$ defined for x and h positive.

(c) $C = 5x^2 + 3(x^2 + 4xh)$ defined for x and h positive.

(d) $V = 720x^2 h$ defined for x and h positive.

12. Cost $= \begin{cases} 12\ell w + 40(w + \ell) \text{ for the rectangular shape} \\ 12\pi r^2 + 40\pi r \text{ for the circular shape} \\ 12(\ell w + \dfrac{\pi w^2}{8}) + 20(2(w + \ell) + \dfrac{\pi w}{2}) \text{ for the last shape} \end{cases}$

defined for w, ℓ and r positive.

13. (a) $P(t) = 100(2^t)$ defined for t positive.

(b) $P(t) = 500(2^{\frac{t}{4}})$ defined for t positive.

(c) $P(t) = 1000(1.1^t)$ defined for t positive.

14. $x = 0$

15. $x = -4$ and $x = -2$

16. $x = 4$

17. $f(x)$ is defined for all real numbers x.

18. $f(x)$ is undefined (as a real-valued function) for $0 \le x \le 1$

19. $f(x)$ is defined for all real numbers x.

20. $f(x)$ is defined for any real number and is zero at $x = 0$ and $x = 7$. This function is positive if $x < 0$ and if $7 < x$.

21. $f(x)$ is defined for any real number and is zero at $x = -3$ and $x = 2$. This function is positive if $-3 < x < 2$.

22. $f(x)$ is undefined at $x = 3$ and is zero at $x = -2$. This function is positive if $x < -2$ and if $3 < x$.

23. $f(x)$ is defined for any real number and is zero at $x = -4, 2$ and 5. This function is positive if $-4 < x < 2$ and if $5 < x$.

24. Intersect at $(-\dfrac{1}{2}, -2)$ and $(1, 1)$.

25. Intersect at $(-2, 2)$ and $(1, 5)$.

26. Intersect at $(5, 3)$.

27. $f(x)g(x) = 3(x + 1), f(g(x)) = \dfrac{3}{x^2 + x}, g(f(x)) = \dfrac{9}{x^2} + \dfrac{3}{x}$,

   $f(1) = 3, g(1) = 2, f(1)g(1) = 6, f(g(1)) = \dfrac{3}{2}, g(f(1)) = 12$

28. $f(x)g(x) = \dfrac{x - 1}{2}, f(g(x)) = \dfrac{x + 1}{2x + 8}, g(f(x)) = \dfrac{5x + 7}{2x + 4}$,

   $f(3) = \dfrac{1}{5}, g(3) = 5, f(3)g(3) = 1, f(g(3)) = \dfrac{2}{7}, g(f(3)) = \dfrac{11}{5}$

29. $f(g(x)) = \dfrac{1}{x + h}$

30. $f(g(x)) = \sqrt{x + h} + 3$

31. $f(g(x)) = (x + h)^2 + (x + h) - 4$

32. $f(g(x)) = \dfrac{x + h + 1}{x + h - 2}$

33. $\dfrac{f(x + h) - f(x)}{h} = 2x + h$

34. $\dfrac{f(x + h) - f(x)}{h} = \dfrac{2}{(x + 2)(x + h + 2)}$

2 Exercises (page 7)

1. 4

2. $2\ln(x-2)$

3. $(x+3)^2$

4. $2x+3$

5. $x = \dfrac{\ln(4)}{3}$

6. $x = \pm\sqrt{\ln(17)} \approx \pm 2.833$

7. $x = \dfrac{-1 \pm \sqrt{1+4e^2}}{2}$, which is $x \approx 2.264$ or -3.264

8. $x = \pm e^{7/6} \approx \pm 3.211$

9. $x = 4$

10. $x = \ln\left(\dfrac{71}{8}\right) \approx 2.183$

11. (a) $598.34
    (b) $598.61
    (c) $1344.89
    (d) $11,051.68
    (e) $6795.70

12. $t = \dfrac{\ln(2)}{0.05} = 13.86$ years

13. 34.65% and 10.98%

14. (a) $P(t) = 158.423e^{.0279t}$
    (b) $P(2020) = 209.404684$ or $209,404,684$
    (c) $t = 8.36$ years from 2010
    (d) $t = 24.84$ years from 2010

15. (a) $P(t) = 0.35417e^{0.00775t}$
    (b) $P(2020) = 0.446874$ or $446,874$
    (c) $t = 89.44$ years from 1990
    (d) $t = 787.52$ years from 1990

3 Exercises (page 9)

1. $\dfrac{13}{5}$ 3. $\dfrac{1}{4}$ 5. $\dfrac{7}{4}$ 6. $-\dfrac{1}{9}$

2. 0   4. 0   7. 1

8. 0

9. $\dfrac{1}{2}$

10. $-\infty$

11. ∞

12. Degree of the numerator is 26; the denominator is of degree 5. The limit is $-\infty$.

13. Degree of the numerator is 26; the denominator is of degree 5. The limit is ∞.

14. Degree of the numerator and denominator is 26. The limit is $\dfrac{1}{2}$.

15. Degree of the numerator is 4; the denominator is of degree 15. The limit is 0.

16. (a) The limit will be zero.

    (b) The limit is either infinity or negative infinity.

    (c) The limit will be the quotient of the coefficient of the term with the highest power for $p(x)$ and the coefficient of the term with the highest power for $q(x)$.

17. (a) $\lim\limits_{x \to \infty} f(x) = 0 \qquad \lim\limits_{x \to -\infty} f(x) = 0$

    (b) No vertical asymptotes; Horizontal asymptote $y = 0$

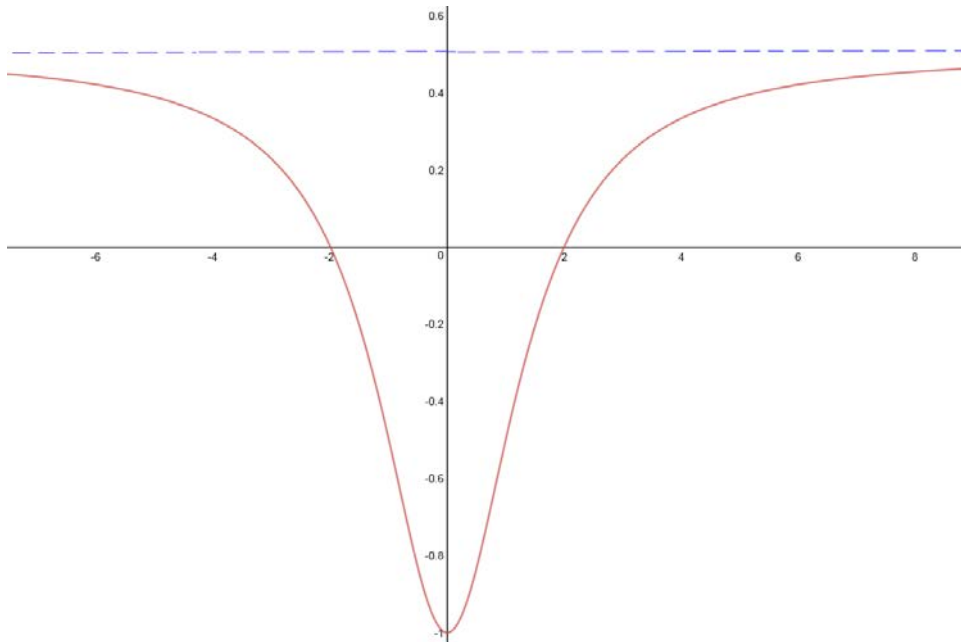   (c) Intercepts at $\left(0, \dfrac{3}{4}\right)$ and $(-3, 0)$

    (d)
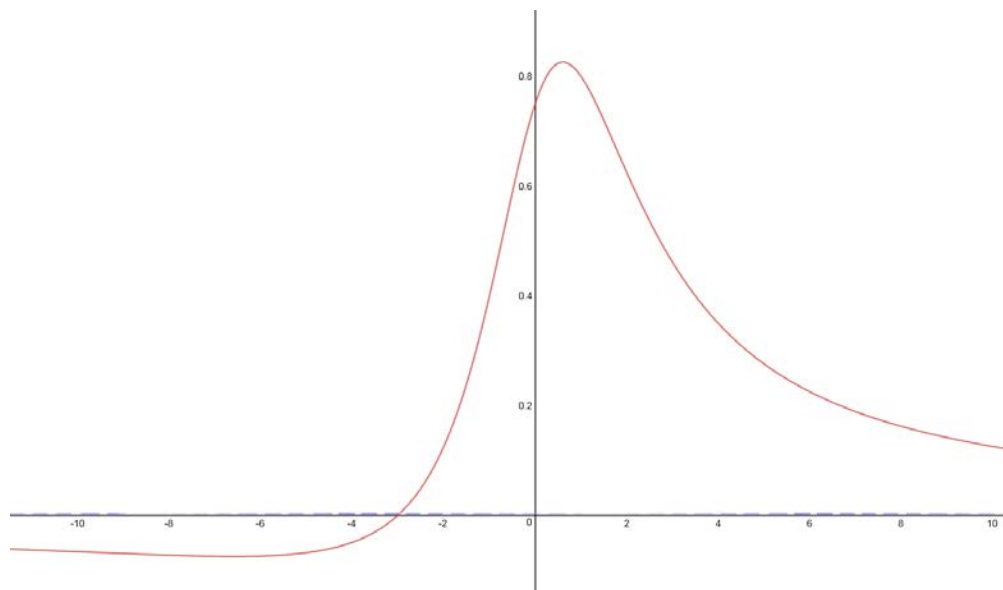
18. (a) $\lim\limits_{x \to \infty} f(x) = \infty$ $\lim\limits_{x \to -\infty} f(x) = -\infty$

    (b) No vertical or horizontal asymptotes.
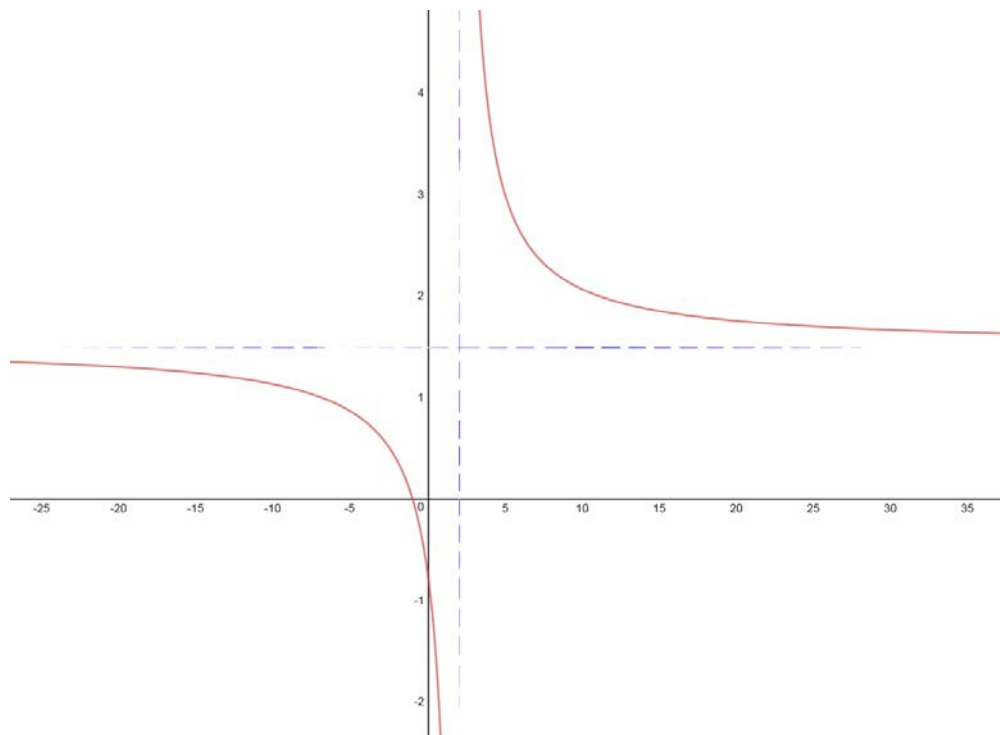
    (c) Intercepts at $(0,0)$ and $(\pm 3, 0)$

    (d)
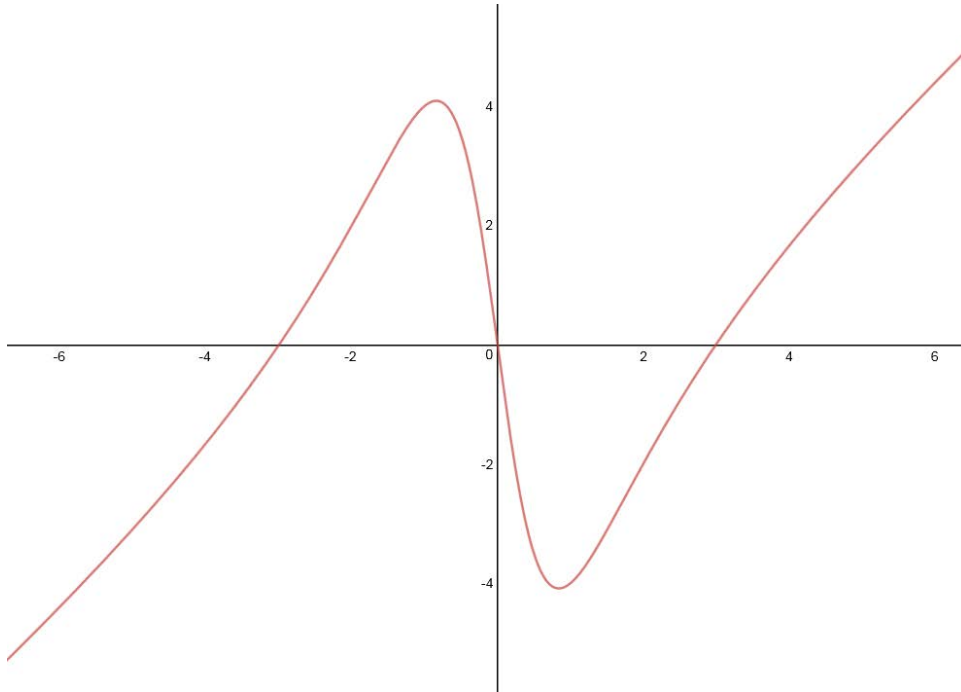
19. (a) $\lim\limits_{x \to \infty} f(x) = \dfrac{1}{2}$ $\lim\limits_{x \to -\infty} f(x) = \dfrac{1}{2}$

    (b) No vertical asymptotes; Horizontal asymptote $y = \dfrac{1}{2}$

    (c) Intercepts at $(\pm 2, 0)$

    (d)

20. (a) $\lim\limits_{x \to \infty} f(x) = \dfrac{3}{2}$ $\lim\limits_{x \to -\infty} f(x) = \dfrac{3}{2}$

     (b) $f(x)$ is undefined at $x = 2$ $\lim\limits_{x \to 2^-} f(x) = -\infty$ $\lim\limits_{x \to 2^+} f(x) = \infty$

          Vertical asymptote $x = 2$; Horizontal asymptote $y = \dfrac{3}{2}$

     (c) Intercepts at $\left(0, -\dfrac{3}{4}\right)$ and $(-1, 0)$

     (d)

21. (a) $\lim\limits_{x \to \infty} f(x) = \dfrac{1}{2}$ $\lim\limits_{x \to -\infty} f(x) = \dfrac{1}{2}$

     (b) $f(x)$ is undefined at $x = -2, 3$
          $\lim\limits_{x \to -2^-} f(x) = \infty$ $\lim\limits_{x \to -2^+} f(x) = -\infty$
          $\lim\limits_{x \to 3^-} f(x) = -\infty$ $\lim\limits_{x \to 3^+} f(x) = +\infty$

          Vertical asymptotes $x = -2$ and $x = 3$; Horizontal asymptote $y = \dfrac{1}{2}$

     (c) Intercepts at $\left(0, -\dfrac{1}{12}\right)$ and $(1, 0)$

(d)
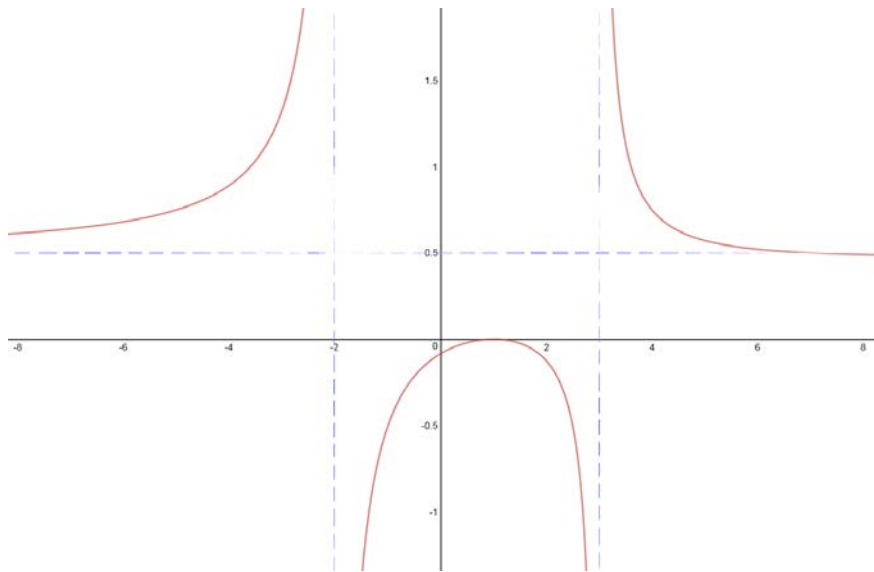
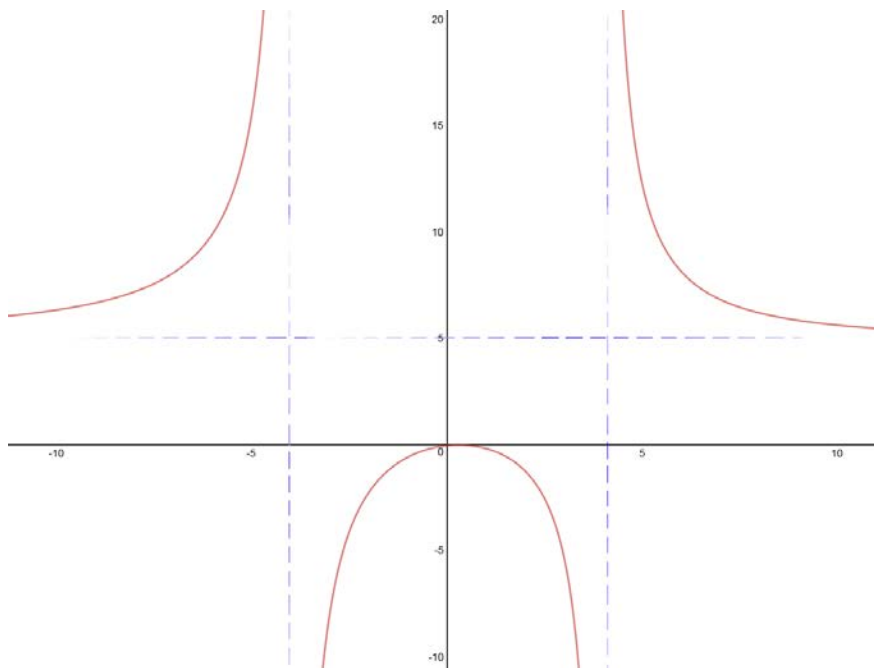22. (a) $\lim\limits_{x\to\infty} f(x) = 5$ $\lim\limits_{x\to-\infty} f(x) = 5$

(b) $f(x)$ is undefined at $x = \pm 4$
$$\lim_{x\to-4^-} f(x) = \infty \qquad \lim_{x\to-4^+} f(x) = -\infty$$
$$\lim_{x\to4^-} f(x) = -\infty \qquad \lim_{x\to4^+} f(x) = +\infty$$
Vertical asymptotes $x = \pm 4$; Horizontal asymptote $y = 5$

(c) Intercept at $\left(0, -\dfrac{1}{16}\right)$

(d)

4 Exercises (page 11)

1. $f(g(x)) = \dfrac{1}{x+h} + \sqrt{x+h} + 3$

2. $f(g(x)) = x^2 + 2xh + h^2 + x + h - 4$

3. $f(g(x)) = \dfrac{x+h+1}{x+h-2}$

4. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{(x+h+4) - (x+4)}{h} = 1$

5. $\dfrac{f(x+h) - f(x)}{h} = 3x^2 + 3xh + h^2 - 1$

6. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{3}{(1 - 2x - 2h)(1 - 2x)}$

7. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\ln(x+h) - \ln(x)}{h}$

8. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{e^{2x}(e^{2h} - 1)}{h}$

9. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{x^2 + xh + 6x + 3h + 1}{(x+h+3)(x+3)}$

10. $f'(x) = 6x - 1$

11. $f'(x) = \dfrac{3}{(1-x)^2}$

12. $f'(x) = \dfrac{2}{\sqrt{2+4x}}$

13. $f'(x) = \dfrac{1}{(1+x)^2}$

14. $s'(0) = 0$ m/s and $s'(4) = 24$ m/s

15. $s'(1) = -\dfrac{1}{18}$ m/s

16. (a) $t(x) = 9x - 17$

    (b) $f(4.1) = 19.91$, $t(4.1) = 19.9$, $f(4.2) = 20.84$, $t(4.2) = 20.8$, $f(6) = 41$, $t(6) = 37$

17. $f'(x) = 20x^4 - 6x^2$

18. $f'(x) = 24x^{23} - \dfrac{2}{3}x^{-\frac{1}{3}} + \dfrac{1}{x^2}$

19. $f'(x) = -x^{-3/2}$

20. $f'(x) = 4.6x^{3.6}$

21. $f'(x) = 60x^2 - 72x + 47$
$f''(x) = 120x - 72$
$f'''(x) = 120$

22. $f'(x) = -4x^{-5} - \dfrac{1}{2}x^{-\frac{1}{2}}$
$f''(x) = 20x^{-6} + \dfrac{1}{4}x^{-\frac{3}{2}}$
$f'''(x) = -120x^{-7} - \dfrac{3}{8}x^{-\frac{5}{2}}$

5 Exercises (page 13)

1. Local and global min at $x = 0.5$ and global max at $x = 7$.

2. Local and global min at $x = \pm\dfrac{1}{\sqrt{2}}$. Local max at $x = 0$, global max at $x = \pm 2$.

3. Local and global min at $x = 4$, global max at $x = 10$.

4. Local max at $x = 0$, local min at $x = \dfrac{500}{3}$. Global max at $x = 500$.

5. Maximum of $xy = 36$ when $x = y = 6$.

6. The area $A(w) = 1000w - 2w^2$ is maximized when $w = 250$ m and $\ell = 500$ m.

7. $w = 4$ m and $\ell = 8$ m

8. $v = \pi r(50 - r^2)$, $r = \sqrt{\dfrac{50}{3}} \approx 4.082$ cm

9. $w = \sqrt[3]{50} \approx 3.68$ cm and $h \approx 7.37$ cm

10. The same solution as the previous question; $3.68 \times 3.68 \times 7.37$ meters.

11. $A = 8w + 57 + \dfrac{100}{w}$, maximized at $w = \sqrt{12.5} \approx 3.54$ cm

12. Squares of side length 3 cm.

13. $C(r) = 0.12\pi r^2 + \dfrac{4.2\pi}{r}$, and minimized at $r = \sqrt[3]{17.5} \approx 2.6$ cm.

14. (a) Inventory Cost $= \$16t + \$4\left(\dfrac{x}{2}\right)$

    (b) The cost is minimized at $t = 10$ and $x = 80$.

6 Exercises (page 15)

1. $f'(x) = -27(4x^{34} + 2x^{21} + x)^{-28}(136x^{33} + 42x^{20} + 1)$

2. $f'(x) = -\dfrac{3}{2}(x^5 + 3x^3 - 2)^{-3/2}(5x^4 + 9x^2)$

3. $f'(x) = 12(2 + \dfrac{1}{3x})^{11}(-\dfrac{1}{3}x^{-2})$

4. $f'(x) = 3(1 + \sqrt{4x})^2(\frac{1}{2}(4x)^{-1/2}(4))$

5. $\frac{df}{dt} = (3(\sqrt{t} + t - 30)^2 + 6(\sqrt{t} + t - 30))(\frac{1}{2\sqrt{t}} + 1)$

6. $\frac{df}{dt} = (34 - \frac{1}{2\sqrt{35t + \frac{2}{t}}})(35 - \frac{2}{t^2})$

7. $\frac{dy}{dx} = \frac{1 - 2x}{2y - 1}$ and $\frac{dy}{dx} = -\frac{1}{3}$ at $x = 1$ and $y = 2$. An equation for the tangent line is
   $y - 2 = -\frac{1}{3}(x - 1)$.

8. $\frac{dy}{dx} = \frac{(x + y)^2 - 1}{1 + 2y(x + y)^2}$ and $\frac{dy}{dx} = 8/73$ at $x = -1$ and $y = 4$. An equation for the tangent line
   is $y - 4 = \frac{8}{73}(x + 1)$.

9. 50π m^2/hour

10. At 10 cm, $\frac{dr}{dt} = \frac{\pi}{80}$ cm/min; at 5cm, $\frac{dr}{dt} = \frac{\pi}{20}$ cm/min

11. $\frac{4}{3}$ m/s

12. (a) $\frac{dV}{dt} = 0.5\pi$ cm/sec

    (b) $\frac{dh}{dt} = -0.02$ cm/sec

13. 0.08 cm/s, 0.03 cm/s

14. 127.58 km/h

15. 576.17 km/h

7 Exercises (page 17)

1. $f'(x) = \frac{-8x^3}{(2x^4 - 3)^2}$

2. $f'(x) = \frac{-10}{(2x - 3)^2}$

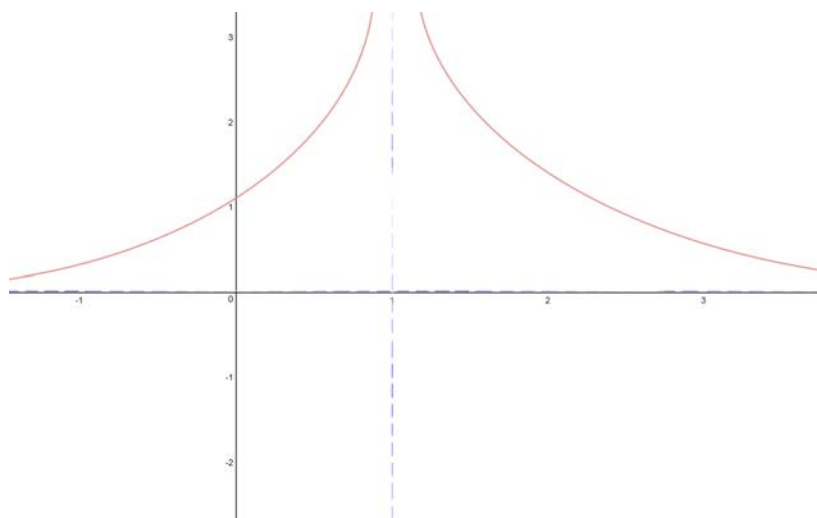3. $f'(x) = -(2x^4 - 3)^{-2}(8x^3)$

4. $f'(x) = -5(2x - 3)^{-2}(2)$

5. $f'(x) = (5x^4 - 20x^3 + 2x)(x^6 - 4x^4 + x + 2) + (x^5 - 5x^4 + x^2 + 4)(6x^5 - 16x^3 + 1)$

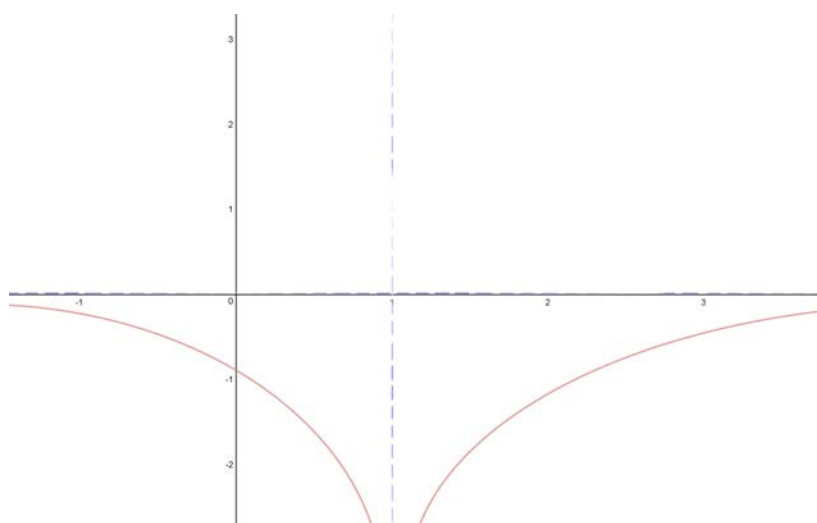6. $f'(x) = 23(3x + 4)^{22}(3)(3x - 3)^{12} + (3x + 4)^{23}(12)(3x - 3)^{11}(3)$

7. $f'(x) = \frac{(x^2 + 4) - (x + 2)(2x)}{(x + 2)^2}$

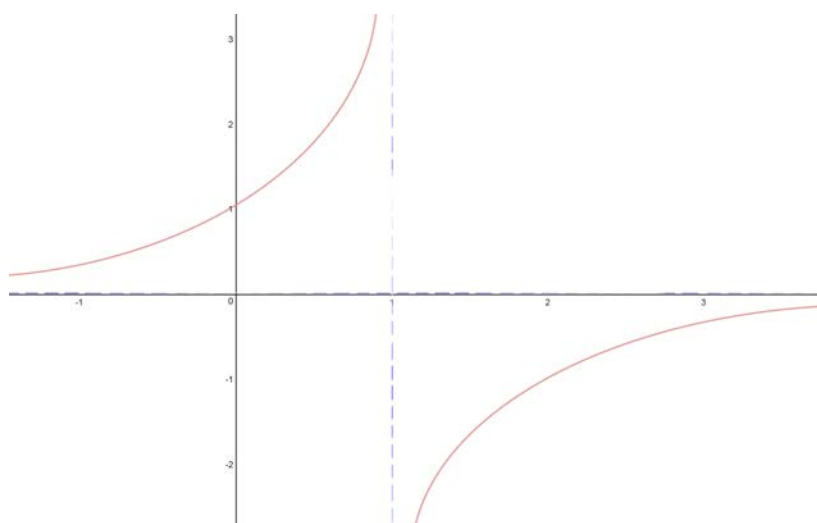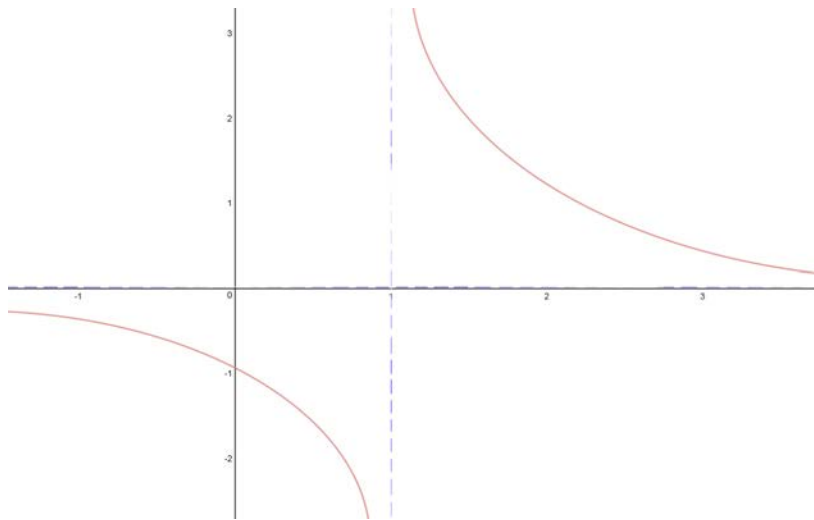8. $f'(x) = \frac{[2(x + 1)(3x - 1)^3 + (x + 1)^2(3)(3x - 1)^2(3)](x + 2) - (x + 1)^2(3x - 1)^3}{(x + 2)^2}$
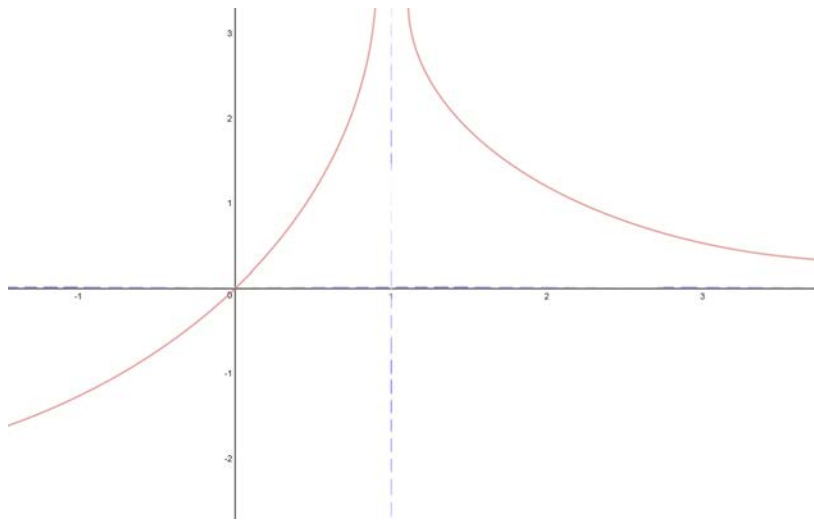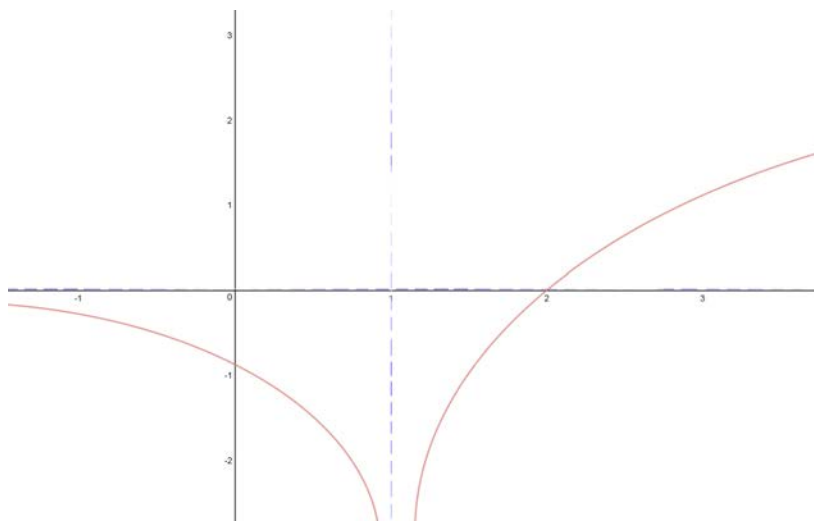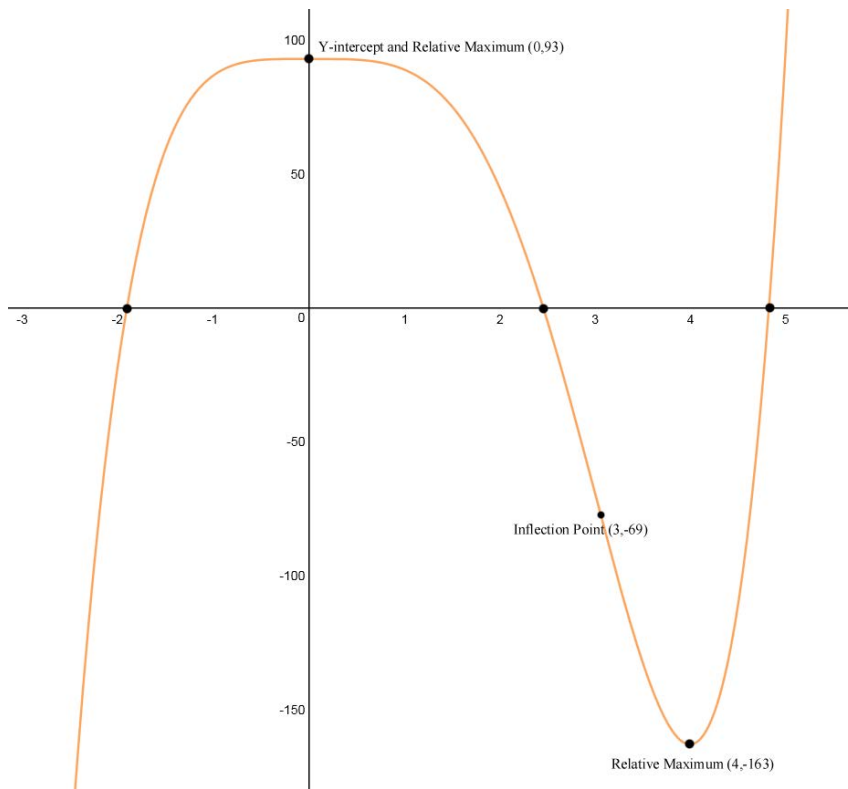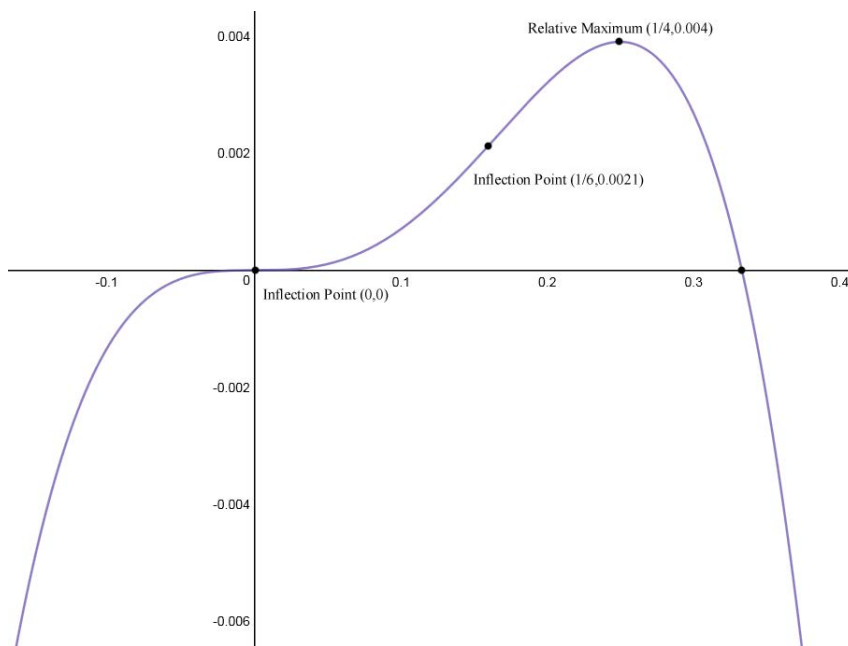
9.

10.

11.

12.

13.

14.

15. Domain $\mathbb{R} = (-\infty, \infty)$; y-int$= (0, 93)$; Increasing on: $(-\infty, 0) \cup (4, \infty)$; Decreasing on: $(0, 4)$; Relative maximum: $f(0) = 93$; Relative minimum: $f(4) = -163$; Concave upward on: $(3, \infty)$; Concave downward on: $(-\infty, 3)$; Inflection point: $(3, -69)$.


16. Domain $\mathbb{R} = (-\infty, \infty)$; y-int$= 0$; x-int$= 0, 1/3$; Increasing on: $(-\infty, 1/4)$; Decreasing on: $(1/4, \infty)$; Relative maximum: $f(1/4) = 0.004$; Concave upward on: $(0, 1/6)$; Concave downward on: $(-\infty, 0) \cup (1/6, \infty)$; Inflection point: $(0, 0)$ and $(1/6, 0.0021)$.

17. Domain $\mathbb{R}\backslash\{-2\}$; y-int$= 0$; x-int$= 0$ Vertical asymptote $x = -2$; Increasing on: $(-\infty, -4) \cup (0, \infty)$; Decreasing on: $(-4, 0)$; Relative maximum: $f(-4) = -8$; Relative minimum: $f(0) = 0$; Concave upward on: $(-2, \infty)$; Concave downward on: $(\infty, -2)$; No inflection points.
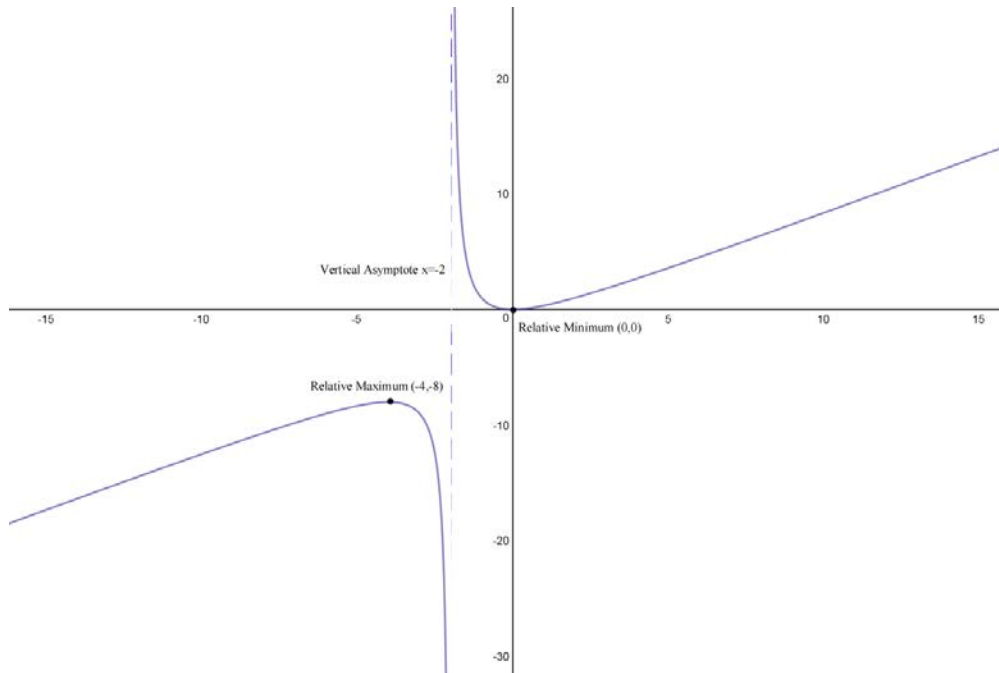
18. Domain $\mathbb{R}$; y-int$= -3/7$; x-int$= 3$ Horizontal asymptote $y = 0$; Increasing on: $(-1, 7)$; Decreasing on: $(-\infty, -1) \cup (7, \infty)$; Relative maximum: $f(7) = 0.071$; Relative minimum: $f(-1) = -0.5$; Concave upward on: $(-2.537, 0.769)$; Concave downward on: $(-\infty, -2.537) \cup (0.769, \infty)$; Inflection points: $(-2.537, -0.412)$ and $(0.769, -0.293)$. Note: Precise locations require using equation solver tool.
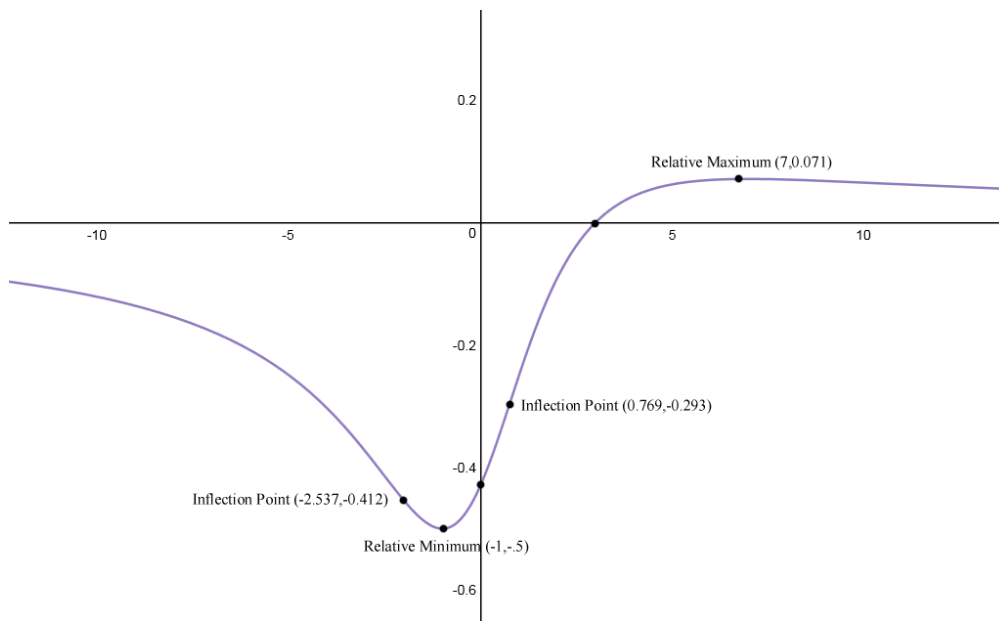
19. Domain $\mathbb{R}$; y-int$= -9$; x-int$= \pm 3$; Horizontal asymptote $y = 1$; Increasing on: $(0, \infty)$; Decreasing on: $(-\infty, 0)$; Relative minimum: $f(0) = -9$; Concave upward on: $\left(-\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$; Concave downward on: $\left(-\infty, -\dfrac{1}{\sqrt{3}}\right) \cup \left(\dfrac{1}{\sqrt{3}}, \infty\right)$; Inflection points: $\left(-\dfrac{1}{\sqrt{3}}, -6.5\right)$ and $\left(\dfrac{1}{\sqrt{3}}, -6.5\right)$.
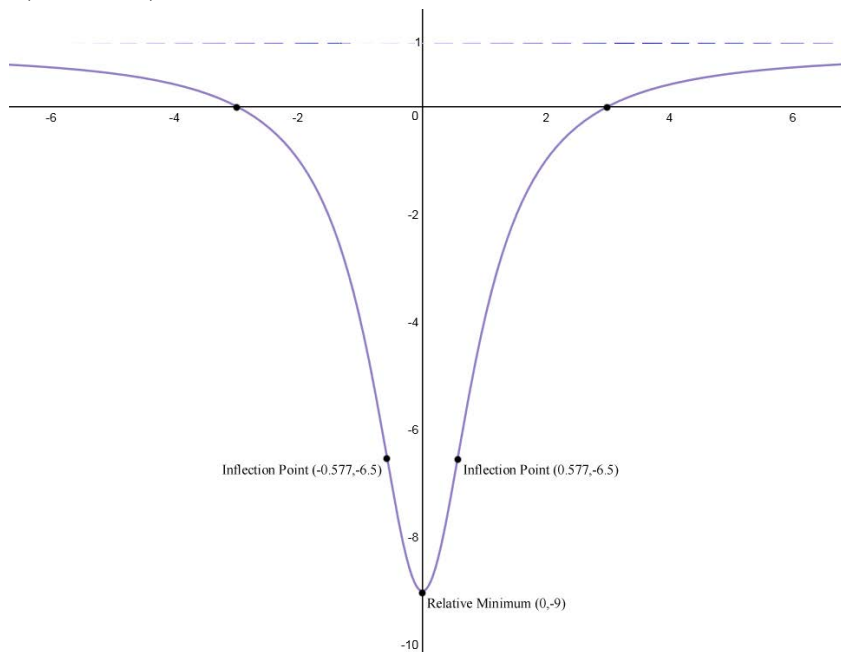
20. Domain $\mathbb{R} \backslash (\{-2\} \cup \{2\})$; y-int$= 3$; x-int$= -3, 4$; Vertical asymptote $x = -2$ and $x = 2$; Horizontal asymptote $y = 1$; Increasing on: $(0, \infty)$; Decreasing on: $(-\infty, 0)$; Relative minimum: $f(-0.254, 2.97)$; Concave upward on: $(-2, 2)$; Concave downward on: $(-\infty, -2) \cup (2, \infty)$; No inflection point.
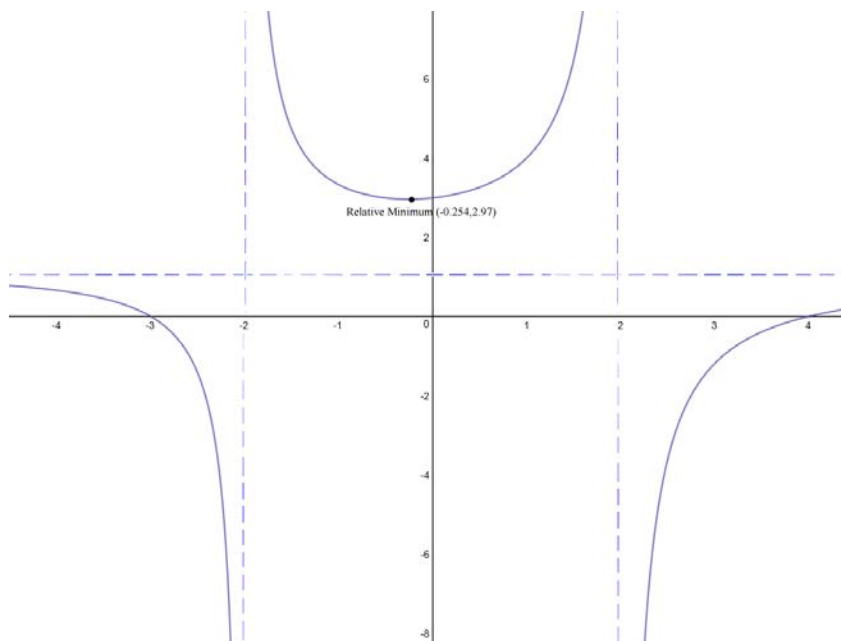
8 Exercises (page)

1. $f'(x) = e^{4x^{23}-x^4}(92x^{22} - 4x^3)$

2. $f'(x) = e^{2x} + 2xe^{2x}$

3. $f'(x) = \dfrac{-xe^{-x}}{(x+1)^2}$

4. $f'(x) = 4(x^3 + e^x + 3)^3(3x^2 + e^x)$

5. $f'(x) = \dfrac{4x}{x^2 - 4}$

6. $f'(x) = \dfrac{1}{x}$

7. $f'(x) = \ln(x) + 1$

8. $f'(x) = \dfrac{3(\ln(x) - 5)^2}{x}$

9. $f'(x) = \dfrac{(2x - 1)\ln(x) - (x - 1)}{\ln(x)^2}$

10. $f'(x) = \ln(x) + 1 - e^{2x}(2x^2 - 1)$

11. $\dfrac{dy}{dx} = x^{3x}(3x\ln(x) + 3x + 1)$

12. $\dfrac{dy}{dx} = (2x^2 - 4)^{\frac{1}{x}}\left(\dfrac{4}{x^2 - 2} - \dfrac{\ln(2x^2 - 4)}{x^2}\right)$

13.   (a) $w(t) = 50(3^t)$

      (b) $w(6) = 36,450$ (yuck!)

      (c) $w'(0) = 54.9$ worms per week

      (d) $w'(6) = 40,044$ worms per week

14. Domain $\mathbb{R}$; y-int$= 10/101$; Horizontal asymptotes $y = 0$ and $y = 10$; Increasing on: $(-\infty, \infty)$; Concave upward on: $(-\infty, \ln 100)$; Concave downward on: $(\ln 100, \infty)$; Inflection point: $(\ln 100, 5)$.
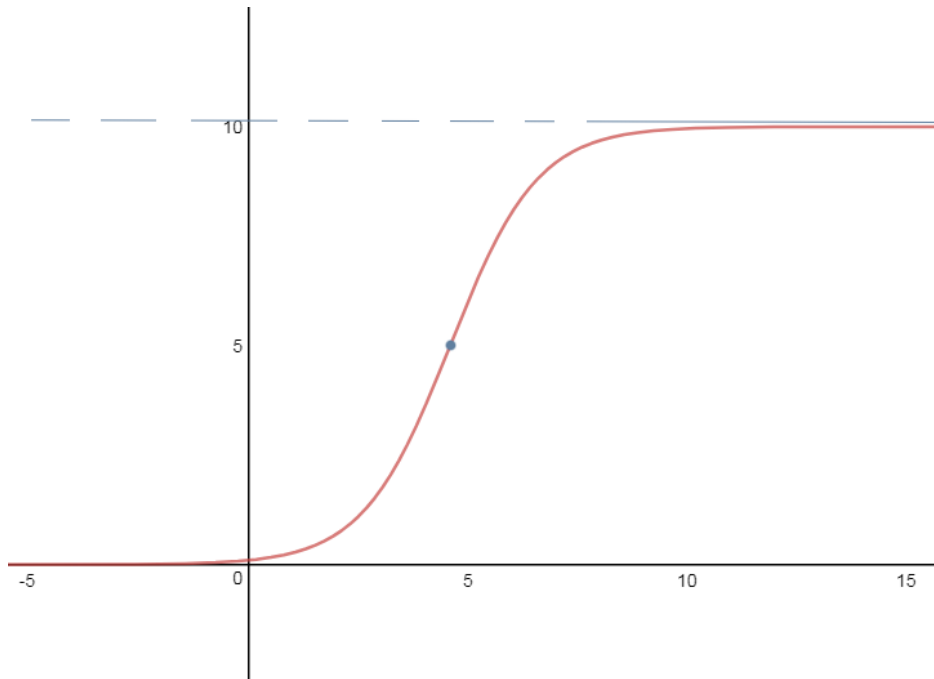
15. Domain $(0, \infty)$; x-int$= 1$; Increasing on: $\left(\dfrac{1}{e}, \infty\right)$; Decreasing on: $\left(0, \dfrac{1}{e}\right)$; Relative minimum: $f\left(\dfrac{1}{e}\right) = -\dfrac{1}{e}$; Concave upward on: $(0, \infty)$; No inflection points.
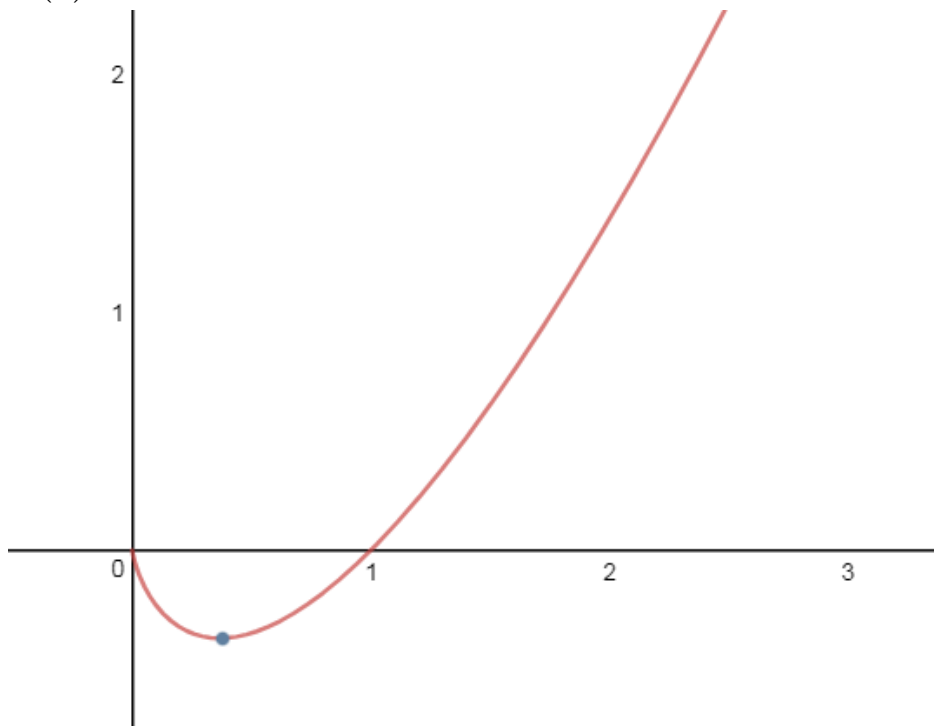
16. Domain $\mathbb{R}$; x-int= 1; Increasing on: $(1, \infty)$; Decreasing on: $(0, 1)$; Relative minimum: $f(1) = 0$; Concave upward on: $(0, e)$; Concave downward on: (e, ∞); Inflection point: $(e, 1)$.
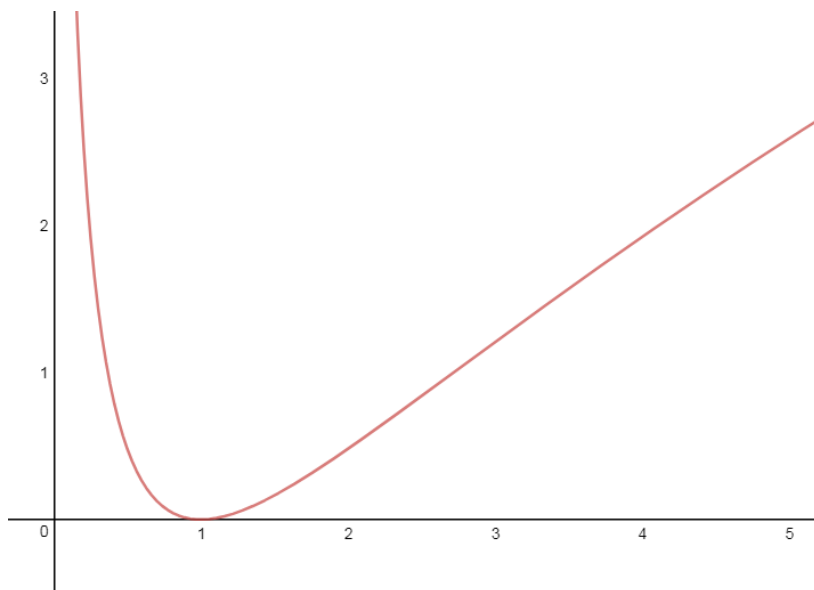
17. Domain $\mathbb{R}$; y-int= 0; x-int= 0; Horizontal asymptote $y = 0$; Increasing on: $(-1, \infty)$; Decreasing on: $(-\infty, -1)$; Relative minimum: $f(-1) = -10/e$; Concave upward on: $(-2, \infty)$; Concave downward on: $(-\infty, -2)$; Inflection point: $\left(-2, -\dfrac{20}{e^2}\right)$.
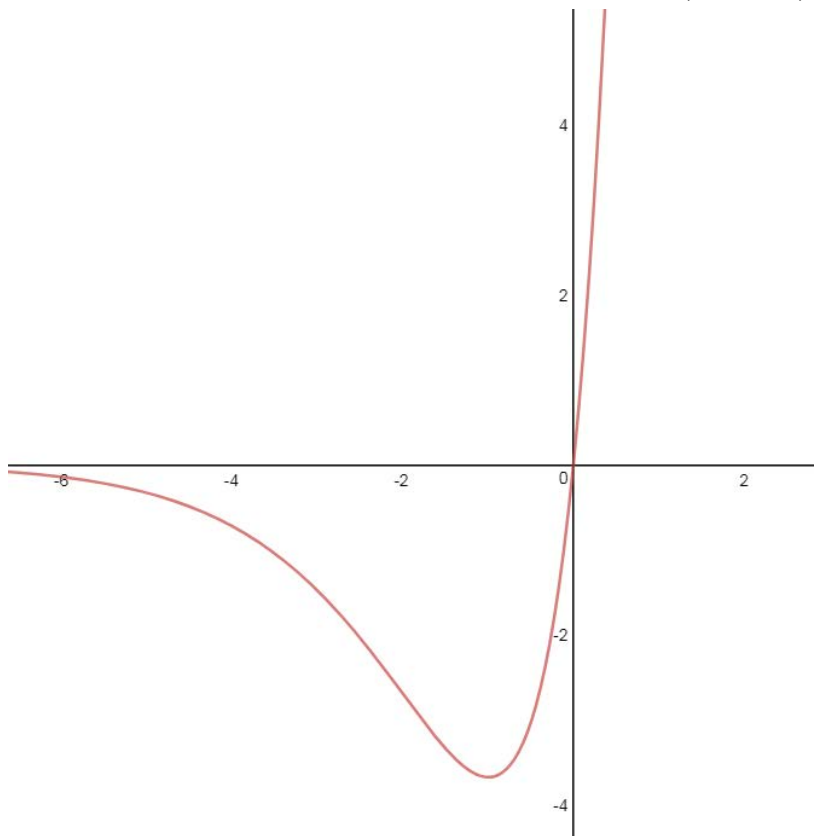
18. max of -0.847 at $x = .707$.

19. max of 0.54 at $x = -2$ and min of 0 at $x = 0$.

20. Minimum of $\$4.08$ at $t = 20$.

21. Maximum effectiveness at $t = 10$ hours.

22. (a) 20

   (b) This is the limit at infinity, so $10,000$.

   (c) 48.3 hours

   (d) $t = 62$

   (e)
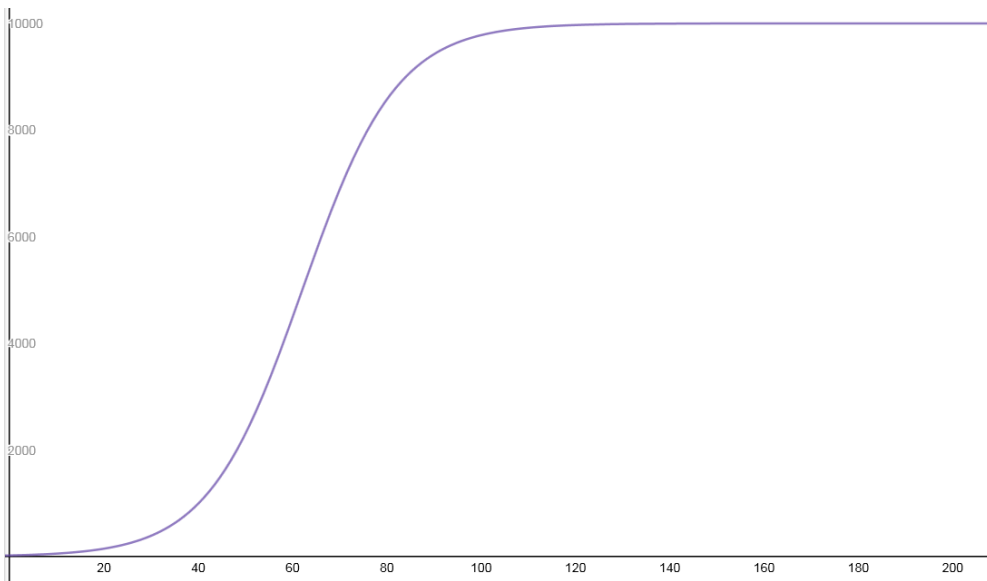
9 Exercises (page 21)

1. $F(x) = x^3 - \dfrac{5}{2}x^2 + 6x + C$

2. $F(x) = \dfrac{1}{2}x^2 - \dfrac{4}{x} + C$

3. $F(x) = \dfrac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + C$

4. $F(x) = \dfrac{3}{5}x^{\frac{5}{3}} - \dfrac{4}{7}x^7 + e^2 x + C$

5. $F(x) = 3\ln(x) + C$

6. $F(x) = \dfrac{1}{2}e^{2x} + C$

7. $\dfrac{4x^3}{3} + 10\sqrt{x} - 4x + C$

8. $\dfrac{x^3}{3} - \dfrac{x^2}{2} - 12x + C$

9. $\dfrac{2\sqrt{2}x^{5/2}}{5} + C$

10. $\dfrac{x^2}{2} + \dfrac{2\sqrt{2}x^{3/2}}{3} + C$

11. 18

13. $(e^{10} - \dfrac{100}{2}) - (e^{-1} - \dfrac{(-1)^2}{2}) = e^{10} - \dfrac{1}{e} - 50.5$

12. -3

14. $\ln(e^2) - \ln(1) = 2 - 0 = 2$

15. $-\dfrac{1}{12}\left(x^3 + 3x^2 + 4\right)^{-4} + C$

16. $\dfrac{1}{3}\left(5x^2 + 2x\right)^{\frac{3}{2}} + C$

17. $\dfrac{1}{40}(4x + 1)^{\frac{5}{2}} - \dfrac{1}{24}(4x + 1)^{\frac{3}{2}} + C$

18. $e^{x^2 - 4} + C$

19. $\ln(2x^2 - 4x) + C$

20. Using $u = x^4 + 9$, integral is $\displaystyle\int_9^{25} u^{\frac{1}{2}}\dfrac{du}{4} = \dfrac{49}{3}$

21. Using $u = x^2$, integral is $= \displaystyle\int_0^{25} e^u\dfrac{du}{2}\dfrac{1}{2}(e^{25} - 1)$

22. Using $u = 2x^2 + 1$, integral is $\displaystyle\int_3^{19} u^{-2}\dfrac{du}{4} = \dfrac{4}{57}$

23. $\dfrac{10\sqrt{5}}{3}$

24. (a)
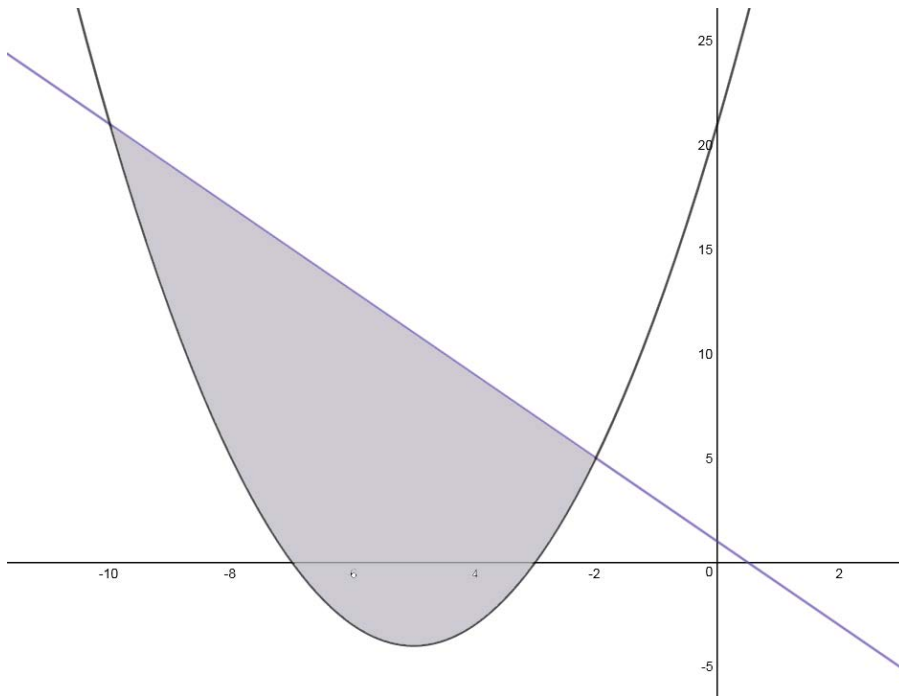
(b) They intersect at $x = -2$ and $x = -10$

(c) $\dfrac{256}{3}$

25. (a)

(b) They intersect at $x = \ln(10) = 2.3$

(c) 28

26. $\dfrac{159}{4}$

27. 39

GNU Free Documentation License

Preamble

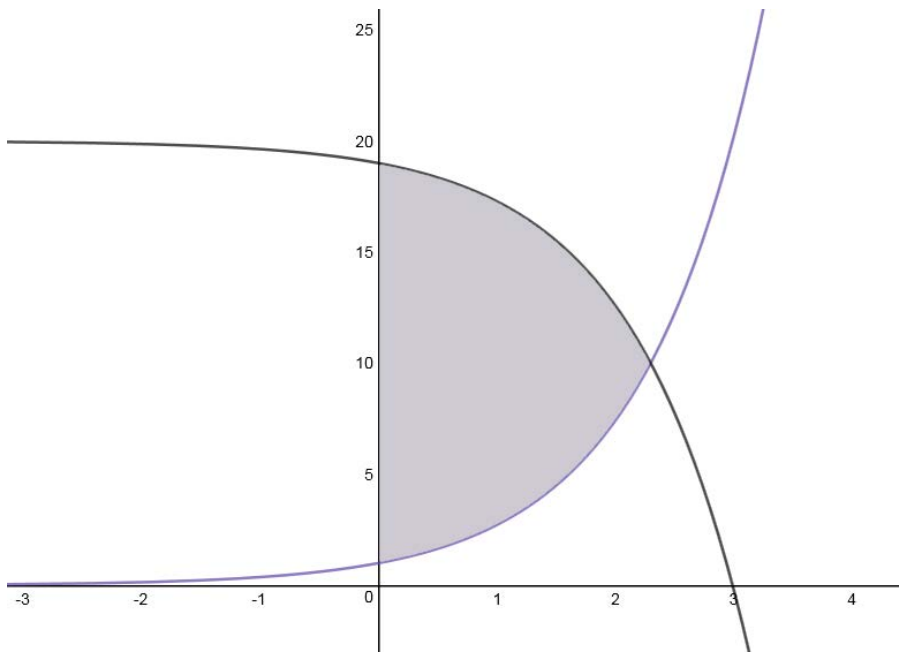The purpose of this License is to make a manual, textbook, or other functional and useful document "free" in the sense of freedom: to assure everyone the effective freedom to copy and redistribute it, with or without modifying it, either commercially or noncommercially. Secondarily, this License preserves for the author and publisher a way to get credit for their work, while not being considered responsible for modifications made by others.

This License is a kind of "copyleft", which means that derivative works of the document must themselves be free in the same sense. It complements the GNU General Public License, which is a copyleft license designed for free software.

We have designed this License in order to use it for manuals for free software, because free software needs free documentation: a free program should come with manuals providing the same freedoms that the software does. But this License is not limited to software manuals; it can be used for any textual work, regardless of subject matter or whether it is published as a printed book. We recommend this License principally for works whose purpose is instruction or reference.

1. APPLICABILITY AND DEFINITIONS

This License applies to any manual or other work, in any medium, that contains a notice placed by the copyright holder saying it can be distributed under the terms of this License. Such a notice grants a world-wide, royalty-free license, unlimited in duration, to use that work under the conditions stated herein. The "**Document**", below, refers to any such manual or work. Any member of the public is a licensee, and is addressed as "**you**". You accept the license if you copy, modify or distribute the work in a way requiring permission under copyright law.

A "**Modified Version**" of the Document means any work containing the Document or a portion of it, either copied verbatim, or with modifications and/or translated into another language.

A "**Secondary Section**" is a named appendix or a front-matter section of the Document that deals exclusively with the relationship of the publishers or authors of the Document to the Document's overall subject (or to related matters) and contains nothing that could fall directly within that overall subject. (Thus, if the Document is in part a textbook of mathematics, a Secondary Section may not explain any mathematics.) The relationship could be a matter of historical connection with the subject or with related matters, or of legal, commercial, philosophical, ethical or political position regarding them.

The "**Invariant Sections**" are certain Secondary Sections whose titles are designated, as being those of Invariant Sections, in the notice that says that the Document is released under this License. If a section does not fit the above definition of Secondary then it is not allowed to be designated as Invariant. The Document may contain zero Invariant Sections. If the Document does not identify any Invariant Sections then there are none.

The "**Cover Texts**" are certain short passages of text that are listed, as Front-Cover Texts or Back-Cover Texts, in the notice that says that the Document is released under this License. A Front-Cover Text may be at most 5 words, and a Back-Cover Text may be at most 25 words.

A "**Transparent**" copy of the Document means a machine-readable copy, represented in a format whose specification is available to the general public, that is suitable for revising the document straightforwardly with generic text editors or (for images composed of pixels) generic paint programs or (for drawings) some widely available drawing editor, and that is suitable for input to text formatters or for automatic translation to a variety of formats suitable for input to text formatters. A copy made in an otherwise Transparent file format whose markup, or absence of markup, has been arranged to thwart or discourage subsequent modification by readers is not Transparent. An image format is not Transparent if used for any substantial amount of text. A copy that is not "Transparent" is called "**Opaque**".

Examples of suitable formats for Transparent copies include plain ASCII without markup, Texinfo input format, LaTeX input format, SGML or XML using a publicly available DTD, and standard-conforming simple HTML, PostScript or PDF designed for human modification. Examples of transparent image formats include PNG, XCF and JPG. Opaque formats include proprietary formats that can be read and edited only by proprietary word processors, SGML or XML for which the DTD and/or processing tools are not generally available, and the machine-generated HTML, PostScript or PDF produced by some word processors for output purposes only.

The "**Title Page**" means, for a printed book, the title page itself, plus such following pages as are needed to hold, legibly, the material this License requires to appear in the title page. For works in formats which do not have any title page as such, "Title Page" means the text near the most prominent appearance of the work's title, preceding the beginning of the body of the text.

The "**publisher**" means any person or entity that distributes copies of the Document to the public.

A section "**Entitled XYZ**" means a named subunit of the Document whose title either is precisely XYZ or contains XYZ in parentheses following text that translates XYZ in another language. (Here XYZ stands for a specific section name mentioned below, such as "**Acknowledgements**", "**Dedications**", "**Endorsements**", or "**History**".) To "**Preserve the Title**" of such a section when you modify the Document means that it remains a section "Entitled XYZ" according to this definition.

The Document may include Warranty Disclaimers next to the notice which states that this License applies to the Document. These Warranty Disclaimers are considered to be included by reference in this License, but only as regards disclaiming warranties: any other implication that these Warranty Disclaimers may have is void and has no effect on the meaning of this License.

2. VERBATIM COPYING

You may copy and distribute the Document in any medium, either commercially or noncommercially, provided that this License, the copyright notices, and the license notice saying this License applies to the Document are reproduced in all copies, and that you add no other conditions whatsoever to those of this License. You may not use technical measures to obstruct or control the reading or further copying of the copies you make or distribute. However, you may accept compensation in exchange for copies. If you distribute a large enough number of copies you must also follow the conditions in section 3.

You may also lend copies, under the same conditions stated above, and you may publicly display copies.

3. COPYING IN QUANTITY

If you publish printed copies (or copies in media that commonly have printed covers) of the Document, numbering more than 100, and the Document's license notice requires Cover Texts, you must enclose the copies in covers that carry, clearly and legibly, all these Cover Texts: Front-Cover Texts on the front cover, and Back-Cover Texts on the back cover. Both covers must also clearly and legibly identify you as the publisher of these copies. The front cover must present the full title with all words of the title equally prominent and visible. You may add other material on the covers in addition. Copying with changes limited to the covers, as long as they preserve the title of the Document and satisfy these conditions, can be treated as verbatim copying in other respects.

If the required texts for either cover are too voluminous to fit legibly, you should put the first ones listed (as many as fit reasonably) on the actual cover, and continue the rest onto adjacent pages.

If you publish or distribute Opaque copies of the Document numbering more than 100, you must either include a machine-readable Transparent copy along with each Opaque copy, or state in or with each Opaque copy a computer-network location from which the general network-using public has access to download using public-standard network protocols a complete Transparent copy of the Document, free of added material. If you use the latter option, you must take reasonably prudent steps, when you begin distribution of Opaque copies in quantity, to ensure that this Transparent copy will remain thus accessible at the stated location until at least one year after the last time you distribute an Opaque copy (directly or through your agents or retailers) of that edition to the public.

It is requested, but not required, that you contact the authors of the Document well before redistributing any large number of copies, to give them a chance to provide you with an updated version of the Document.

4. MODIFICATIONS

You may copy and distribute a Modified Version of the Document under the conditions of sections 2 and 3 above, provided that you release the Modified Version under precisely this License, with the Modified Version filling the role of the Document, thus licensing distribution and modification of the Modified Version to whoever possesses a copy of it. In addition, you must do these things in the Modified Version:

A. Use in the Title Page (and on the covers, if any) a title distinct from that of the Document, and from those of previous versions (which should, if there were any, be listed in the History section of the Document). You may use the same title as a previous version if the original publisher of that version gives permission.

B. List on the Title Page, as authors, one or more persons or entities responsible for authorship of the modifications in the Modified Version, together with at least five of the principal authors of the Document (all of its principal authors, if it has fewer than five), unless they release you from this requirement.

C. State on the Title page the name of the publisher of the Modified Version, as the publisher.

D. Preserve all the copyright notices of the Document.

E. Add an appropriate copyright notice for your modifications adjacent to the other copyright notices.

F. Include, immediately after the copyright notices, a license notice giving the public permission to use the Modified Version under the terms of this License, in the form shown in the Addendum below.

G. Preserve in that license notice the full lists of Invariant Sections and required Cover Texts given in the Document's license notice.

H. Include an unaltered copy of this License.

I. Preserve the section Entitled "History", Preserve its Title, and add to it an item stating at least the title, year, new authors, and publisher of the Modified Version as given on the Title Page. If there is no section Entitled "History" in the Document, create one stating the title, year, authors, and publisher of the Document as given on its Title Page, then add an item describing the Modified Version as stated in the previous sentence.

J. Preserve the network location, if any, given in the Document for public access to a Transparent copy of the Document, and likewise the network locations given in the Document for previous versions it was based on. These may be placed in the "History" section. You may omit a network location for a work that was published at least four years before the Document itself, or if the original publisher of the version it refers to gives permission.

K. For any section Entitled "Acknowledgements" or "Dedications", Preserve the Title of the section, and preserve in the section all the substance and tone of each of the contributor acknowledgements and/or dedications given therein.

L. Preserve all the Invariant Sections of the Document, unaltered in their text and in their titles. Section numbers or the equivalent are not considered part of the section titles.

M. Delete any section Entitled "Endorsements". Such a section may not be included in the Modified Version.

N. Do not retitle any existing section to be Entitled "Endorsements" or to conflict in title with any Invariant Section.

O. Preserve any Warranty Disclaimers.

If the Modified Version includes new front-matter sections or appendices that qualify as Secondary Sections and contain no material copied from the Document, you may at your option designate some or all of these sections as invariant. To do this, add their titles to the list of Invariant Sections in the Modified Version's license notice. These titles must be distinct from any other section titles.

You may add a section Entitled "Endorsements", provided it contains nothing but endorsements of your Modified Version by various parties—for example, statements of peer review or that the text has been approved by an organization as the authoritative definition of a standard.

You may add a passage of up to five words as a Front-Cover Text, and a passage of up to 25 words as a Back-Cover Text, to the end of the list of Cover Texts in the Modified Version. Only one passage of Front-Cover Text and one of Back-Cover Text may be added by (or through arrangements made by) any one entity. If the Document already includes a cover text for the same cover, previously added by you or by arrangement made by the same entity you are acting on behalf of, you may not add another; but you may replace the old one, on explicit permission from the previous publisher that added the old one.

The author(s) and publisher(s) of the Document do not by this License give permission to use their names for publicity for or to assert or imply endorsement of any Modified Version.

5. COMBINING DOCUMENTS

You may combine the Document with other documents released under this License, under the terms defined in section 4 above for modified versions, provided that you include in the combination all of the Invariant Sections of all of the original documents, unmodified, and list them all as Invariant Sections of your combined work in its license notice, and that you preserve all their Warranty Disclaimers.

The combined work need only contain one copy of this License, and multiple identical Invariant Sections may be replaced with a single copy. If there are multiple Invariant Sections with the same name but different contents, make the title of each such section unique by adding at the end of it, in parentheses, the name of the original author or publisher of that section if known, or else a unique number. Make the same adjustment to the section titles in the list of Invariant Sections in the license notice of the combined work.

In the combination, you must combine any sections Entitled "History" in the various original documents, forming one section Entitled "History"; likewise combine any sections Entitled "Acknowledgements", and any sections Entitled "Dedications". You must delete all sections Entitled "Endorsements".

6. COLLECTIONS OF DOCUMENTS

You may make a collection consisting of the Document and other documents released under this License, and replace the individual copies of this License in the various documents with a single copy that is included in the collection, provided that you follow the rules of this License for verbatim copying of each of the documents in all other respects.

You may extract a single document from such a collection, and distribute it individually under this License, provided you insert a copy of this License into the extracted document, and follow this License in all other respects regarding verbatim copying of that document.

7. AGGREGATION WITH INDEPENDENT WORKS

A compilation of the Document or its derivatives with other separate and independent documents or works, in or on a volume of a storage or distribution medium, is called an "aggregate" if the copyright resulting from the compilation is not used to limit the legal rights of the compilation's users beyond what the individual works permit. When the Document is included in an aggregate, this License does not apply to the other works in the aggregate which are not themselves derivative works of the Document.

If the Cover Text requirement of section 3 is applicable to these copies of the Document, then if the Document is less than one half of the entire aggregate, the Document's Cover Texts may be placed on covers that bracket the Document within the aggregate, or the electronic equivalent of covers if the Document is in electronic form. Otherwise they must appear on printed covers that bracket the whole aggregate.

8. TRANSLATION

Translation is considered a kind of modification, so you may distribute translations of the Document under the terms of section 4. Replacing Invariant Sections with translations requires special permission from their copyright holders, but you may include translations of some or all Invariant Sections in addition to the original versions of these Invariant Sections. You may include a translation of this License, and all the license notices in the Document, and any Warranty Disclaimers, provided that you also include the original English version of this License and the original versions of those notices and disclaimers. In case of a disagreement between the translation and the original version of this License or a notice or disclaimer, the original version will prevail.

If a section in the Document is Entitled "Acknowledgements", "Dedications", or "History", the requirement (section 4) to Preserve its Title (section 1) will typically require changing the actual title.

9. TERMINATION

You may not copy, modify, sublicense, or distribute the Document except as expressly provided under this License. Any attempt otherwise to copy, modify, sublicense, or distribute it is void, and will automatically terminate your rights under this License.

However, if you cease all violation of this License, then your license from a particular copyright holder is reinstated (a) provisionally, unless and until the copyright holder explicitly and finally terminates your license, and (b) permanently, if the copyright holder fails to notify you of the violation by some reasonable means prior to 60 days after the cessation.

Moreover, your license from a particular copyright holder is reinstated permanently if the copyright holder notifies you of the violation by some reasonable means, this is the first time you have received notice of violation of this License (for any work) from that copyright holder, and you cure the violation prior to 30 days after your receipt of the notice.

Termination of your rights under this section does not terminate the licenses of parties who have received copies or rights from you under this License. If your rights have been terminated and not permanently reinstated, receipt of a copy of some or all of the same material does not give you any rights to use it.

10. FUTURE REVISIONS OF THIS LICENSE

The Free Software Foundation may publish new, revised versions of the GNU Free Documentation License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns. See http://www.gnu.org/copyleft/.

Each version of the License is given a distinguishing version number. If the Document specifies that a particular numbered version of this License "or any later version" applies to it, you have the option of following the terms and conditions either of that specified version or of any later version that has been published (not as a draft) by the Free Software Foundation. If the Document does not specify a version number of this License, you may choose any version ever published (not as a draft) by the Free Software Foundation. If the Document specifies that a proxy can decide which future versions of this License can be used, that proxy's public statement of acceptance of a version permanently authorizes you to choose that version for the Document.

11. RELICENSING

"Massive Multiauthor Collaboration Site" (or "MMC Site") means any World Wide Web server that publishes copyrightable works and also provides prominent facilities for anybody to edit those works. A public wiki that anybody can edit is an example of such a server. A "Massive Multiauthor Collaboration" (or "MMC") contained in the site means any set of copyrightable works thus published on the MMC site.

"CC-BY-SA" means the Creative Commons Attribution-Share Alike 3.0 license published by Creative Commons Corporation, a not-for-profit corporation with a principal place of business in San Francisco, California, as well as future copyleft versions of that license published by that same organization.

"Incorporate" means to publish or republish a Document, in whole or in part, as part of another Document.

An MMC is "eligible for relicensing" if it is licensed under this License, and if all works that were first published under this License somewhere other than this MMC, and subsequently incorporated in whole or in part into the MMC, (1) had no cover texts or invariant sections, and (2) were thus incorporated prior to November 1, 2008.

The operator of an MMC Site may republish an MMC contained in the site under CC-BY-SA on the same site at any time before August 1, 2009, provided the MMC is eligible for relicensing.

ADDENDUM: How to use this License for your documents

To use this License in a document you have written, include a copy of the License in the document and put the following copyright and license notices just after the title page:

Copyright © YEAR YOUR NAME. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.3 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

If you have Invariant Sections, Front-Cover Texts and Back-Cover Texts, replace the "with . . . Texts." line with this:

with the Invariant Sections being LIST THEIR TITLES, with the Front-Cover Texts being LIST, and with the Back-Cover Texts being LIST.

If you have Invariant Sections without Cover Texts, or some other combination of the three, merge those two alternatives to suit the situation.

If your document contains nontrivial examples of program code, we recommend releasing these examples in parallel under your choice of free software license, such as the GNU General Public License, to permit their use in free software.